thm_2EdefCNF_2EDEF__SNOC (TMKuxQNs-DuzETfcRGkyaZWmaPBm46ZL3f7b)

October 26, 2020

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{3}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{4}$$

Let $c\_2EdefCNF\_2EUNIQUE : \iota$ be given. Assume the following.

$$c\_2EdefCNF\_2EUNIQUE \in (((2^{(ty\_2Epair\_2Eprod\ ((2^2)^2)\ (ty\_2Epair\_2Eprod\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enu} \tag{5}$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{6}$$

Let $c\_2EdefCNF\_2EDEF : \iota$ be given. Assume the following.

$$c\_2EdefCNF\_2EDEF \in (((2^{(ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ ((2^2)^2)\ (ty\_2Epair\_2Eprod\ (ty\_2Esum\_2Esum\ ty\_2 \tag{7}$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{8}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{9}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{10}$$

Definition 5 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{11}$$

Definition 6 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{12}$$

Definition 7 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{13}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{14}$$

Let $c\_2Elist\_2ESNOC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ESNOC\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{15}$$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ \mathsf{c\_2Enum\_2E0}) = V0m)) \tag{16}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Enum\_2E0})\ V0m) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ \mathsf{c\_2Enum\_2E0}) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)))))))) \tag{17}$$

Assume the following.

$$True \tag{18}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \tag{19}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{21}$$

Assume the following.

$$((\forall V0v \in (2^{ty\_2Enum\_2Enum}).(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ (ap\ c\_2EdefCNF\_2EDEF\ V0v)\ V1n)\ (c\_2Elist\_2ENIL\ (ty\_2Epair\_2Eprod\ ((2^2)^2)\ (ty\_2Epair\_2Eprod\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ 2)\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ 2))))))) \Leftrightarrow True))) \wedge (\forall V2v \in (2^{ty\_2Enum\_2Enum}).(\forall V3n \in ty\_2Enum\_2Enum.(\forall V4x \in (ty\_2Epair\_2Eprod\ ((2^2)^2)\ (ty\_2Epair\_2Eprod\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ 2)\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ 2))).(\forall V5xs \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ ((2^2)^2)\ (ty\_2Epair\_2Eprod\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ 2)\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ 2)))).((p\ (ap\ (ap\ (ap\ c\_2EdefCNF\_2EDEF\ V2v)\ V3n)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ (ty\_2Epair\_2Eprod\ ((2^2)^2)\ (ty\_2Epair\_2Eprod\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ 2)\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ 2))))\ V4x)\ V5xs))) \Leftrightarrow ((p\ (ap\ (ap\ (ap\ c\_2EdefCNF\_2EUNIQUE\ V2v)\ V3n)\ V4x)) \wedge (p\ (ap\ (ap\ (ap\ c\_2EdefCNF\_2EDEF\ V2v)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V3n))\ V5xs))))))))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a)) = \mathsf{c\_2Enum\_2E0}) \wedge (\forall V0h \in A\_27a.(\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ V1t)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1t)))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}).(((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t))))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ V0P\ V3l))))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0x \in A\_27a.((ap\ (ap\ (c\_2Elist\_2ESNOC\ A\_27a)\ V0x)\ (c\_2Elist\_2ENIL\ A\_27a)) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0x)\ (c\_2Elist\_2ENIL\ A\_27a)))) \wedge (\forall V1x \in A\_27a.(\forall V2x\_27 \in A\_27a.(\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2ESNOC\ A\_27a)\ V1x)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2x\_27)\ V3l)) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2x\_27)\ (ap\ (ap\ (c\_2Elist\_2ESNOC\ A\_27a)\ V1x)\ V3l)))))))) \tag{25}$$

Theorem 1

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1x \in (ty\_2Epair\_2Eprod\ ((2^2)^2)\ (ty\_2Epair\_2Eprod\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ 2)\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ 2))).(\forall V2l \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ ((2^2)^2)\ (ty\_2Epair\_2Eprod\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ 2)\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ 2)))).(\forall V3v \in (2^{ty\_2Enum\_2Enum}).((p\ (ap\ (ap\ (ap\ c\_2EdefCNF\_2EDEF\ V3v)\ V0n)\ (ap\ (ap\ (c\_2Elist\_2ESNOC\ (ty\_2Epair\_2Eprod\ ((2^2)^2)\ (ty\_2Epair\_2Eprod\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ 2)\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ 2))))\ V1x)\ V2l))) \Leftrightarrow ((p\ (ap\ (ap\ (ap\ c\_2EdefCNF\_2EDEF\ V3v)\ V0n)\ V2l)) \wedge (p\ (ap\ (ap\ (ap\ c\_2EdefCNF\_2EUNIQUE\ V3v)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ (ap\ (c\_2Elist\_2ELENGTH\ (ty\_2Epair\_2Eprod\ ((2^2)^2)\ (ty\_2Epair\_2Eprod\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ 2)\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ 2))))\ V2l)))\ V1x))))))))$$