

thm_2EdefCNF_2EOKDEF__SNOC (TMPnUn- PCvjmRKxg8BY4UUPSs3gKzCMFihx4)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{3}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{4}$$

Let $c_2EdefCNF_2EOK : \iota$ be given. Assume the following.

$$c_2EdefCNF_2EOK \in ((2^{(ty_2Epair_2Eprod\ ((2^2)^2)})\ (ty_2Epair_2Eprod\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ 2))\ (ty_2Enum_2Enum))) \tag{5}$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \tag{6}$$

Let $c_2EdefCNF_2EOKDEF : \iota$ be given. Assume the following.

$$c_2EdefCNF_2EOKDEF \in ((2^{(ty_2Elist_2Elist (ty_2Epair_2Eprod ((2^2)^2) (ty_2Epair_2Eprod (ty_2Esum_2Esum t$$
 (7)

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum})$$
 (8)

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega})$$
 (9)

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega})$$
 (10)

Definition 5 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega$$
 (11)

Definition 6 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ELENGTH A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist A_27a)})$$
 (12)

Definition 7 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a})$$
 (13)

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a)$$
 (14)

Let $c_2Elist_2ESNOC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ESNOC A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a})$$
 (15)

Definition 8 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E21 2) (\lambda V2t \in 2$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2E_2B V0m) c_2Enum_2E0) = V0m)) \quad (16)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (ap c_2Earithmic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmic_2E_2B (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmic_2E_2B V0m) V1n))) \wedge ((ap (ap c_2Earithmic_2E_2B V0m) (ap c_2Enum_2ESUC V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmic_2E_2B V0m) V1n)))))))))) \quad (17)$$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (21)$$

Assume the following.

$$((\forall V0n \in ty_2Enum_2Enum.((p (ap (ap c_2EdefCNF_2EOKDEF V0n) (c_2Elist_2ENIL (ty_2Epair_2Eprod ((2^2)^2) (ty_2Epair_2Eprod (ty_2Esum_2Esum ty_2Enum_2Enum 2) (ty_2Esum_2Esum ty_2Enum_2Enum 2)))))) \Leftrightarrow True)) \wedge (\forall V1n \in ty_2Enum_2Enum.(\forall V2x \in (ty_2Epair_2Eprod ((2^2)^2) (ty_2Epair_2Eprod (ty_2Esum_2Esum ty_2Enum_2Enum 2) (ty_2Esum_2Esum ty_2Enum_2Enum 2))).(\forall V3xs \in (ty_2Elist_2Elist (ty_2Epair_2Eprod ((2^2)^2) (ty_2Epair_2Eprod (ty_2Esum_2Esum ty_2Enum_2Enum 2) (ty_2Esum_2Esum ty_2Enum_2Enum 2))))).((p (ap (ap c_2EdefCNF_2EOKDEF V1n) (ap (ap (c_2Elist_2ECONS (ty_2Epair_2Eprod ((2^2)^2) (ty_2Epair_2Eprod (ty_2Esum_2Esum ty_2Enum_2Enum 2) (ty_2Esum_2Esum ty_2Enum_2Enum 2)))))) V2x)) \Leftrightarrow ((p (ap (ap c_2EdefCNF_2EOK V1n) V2x)) \wedge (p (ap (ap c_2EdefCNF_2EOKDEF (ap c_2Enum_2ESUC V1n)) V3xs))))))))) \quad (22)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (((ap\ (c_2Elist_2ELENGTH\ A_27a) \\
& \quad (c_2Elist_2ENIL\ A_27a)) = c_2Enum_2E0) \wedge (\forall V0h \in A_27a. (\\
& \quad \forall V1t \in (ty_2Elist_2Elist\ A_27a). ((ap\ (c_2Elist_2ELENGTH \\
& \quad A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V0h)\ V1t)) = (ap\ c_2Enum_2ESUC \\
& \quad (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V1t))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\
& \quad (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\
& \quad A_27a). ((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a. (p\ (ap\ V0P\ (ap\ (ap\ (\\
& \quad c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\
& \quad A_27a). (p\ (ap\ V0P\ V3l))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0x \in A_27a. ((ap\ (ap\ (c_2Elist_2ESNOC \\
& \quad A_27a)\ V0x)\ (c_2Elist_2ENIL\ A_27a)) = (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad A_27a)\ V0x)\ (c_2Elist_2ENIL\ A_27a)))) \wedge (\forall V1x \in A_27a. (\forall V2x.27 \in \\
& \quad A_27a. (\forall V3l \in (ty_2Elist_2Elist\ A_27a). ((ap\ (ap\ (c_2Elist_2ESNOC \\
& \quad A_27a)\ V1x)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2x.27)\ V3l)) = (ap\ (\\
& \quad ap\ (c_2Elist_2ECONS\ A_27a)\ V2x.27)\ (ap\ (ap\ (c_2Elist_2ESNOC\ A_27a) \\
& \quad V1x)\ V3l))))))
\end{aligned} \tag{25}$$

Theorem 1

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1x \in (ty_2Epair_2Eprod \\
& \quad ((2^2)^2)\ (ty_2Epair_2Eprod\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ 2) \\
& \quad (ty_2Esum_2Esum\ ty_2Enum_2Enum\ 2))). (\forall V2l \in (ty_2Elist_2Elist \\
& \quad (ty_2Epair_2Eprod\ ((2^2)^2)\ (ty_2Epair_2Eprod\ (ty_2Esum_2Esum \\
& \quad ty_2Enum_2Enum\ 2)\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ 2))))). ((\\
& \quad p\ (ap\ (ap\ c_2EdefCNF_2EOKDEF\ V0n)\ (ap\ (ap\ (c_2Elist_2ESNOC\ (ty_2Epair_2Eprod \\
& \quad ((2^2)^2)\ (ty_2Epair_2Eprod\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ 2) \\
& \quad (ty_2Esum_2Esum\ ty_2Enum_2Enum\ 2))))\ V1x)\ V2l))) \Leftrightarrow ((p\ (ap\ (ap \\
& \quad c_2EdefCNF_2EOKDEF\ V0n)\ V2l)) \wedge (p\ (ap\ (ap\ c_2EdefCNF_2EOK\ (ap\ (\\
& \quad ap\ c_2Earithmetic_2E_2B\ V0n)\ (ap\ (c_2Elist_2ELENGTH\ (ty_2Epair_2Eprod \\
& \quad ((2^2)^2)\ (ty_2Epair_2Eprod\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ 2) \\
& \quad (ty_2Esum_2Esum\ ty_2Enum_2Enum\ 2))))\ V2l)))\ V1x))))))
\end{aligned}$$