

thm_2Edft_2EDFT__FOLD

(TMN1fVWtoexprLqzSevHGQVm6hAFMaAchvdY)

October 26, 2020

Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define `c_2Ebool_2EF` to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 7 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p (ap P x)))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define `c_2Ebool_2ECOND` to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap$

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let `c_2Elist_2ELIST__TO__SET` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ELIST_TO_SET A_27a \in ((2^{A_27a})^{(ty_2Elist_2Elist A_27a)}) \quad (2)$$

Definition 9 We define `c_2Ebool_2EIN` to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Let `c_2Edft_2EDFT` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Edft_2EDFT A_27a A_27b \in (((((A_27a^{A_27a})^{(ty_2Elist_2Elist A_27b)})^{(ty_2Elist_2Elist A_27b)})^{(A_27a^{A_27a})^{A_27b}}))^{(ty_2Elist_2Elist A_27a)}) \quad (3)$$

Let `c_2Elist_2ENIL` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (4)$$

Definition 10 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_2E_3D_3D_3E V0t) c_Ebool_2E_7E))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (5)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b} A_27a)}) \quad (6)$$

Definition 11 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS_prod A_27a A_27b) x y))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (7)$$

Definition 12 We define $c_2EdirGraph_2EParents$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0G \in ((ty_2Elist_2Elist A_27a A_27b) G))$

Definition 13 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21 2) (\lambda V2t \in 2.(ap (c_Ebool_2E_21 2) t2)) t1)))$

Definition 14 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2Epred_set_2EGSPEC A_27a) x s))$

Definition 15 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_Ebool_2EF)$.

Definition 16 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_Ebool_2E_21 2) s))$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EAPPEND A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{(ty_2Elist_2Elist A_27a)}) \quad (8)$$

Let $c_2Elist_2EFOLDR : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Elist_2EFOLDR A_27a A_27b \in (((A_27b^{(ty_2Elist_2Elist A_27a)})^{A_27b})^{((A_27b^{A_27b})^{A_27a})}) \quad (9)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (10)$$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (16)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (21)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\ A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF) \\ V0t1)\ V1t2) = V1t2)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))))) \Leftrightarrow \\ ((\forall V3x \in A_27a. (p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A_27a. (p\ (\\ ap\ V1Q\ V4x)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ 2^{A_27a}). (((p\ V0P) \wedge (\forall V2x \in A_27a. (p\ (ap\ V1Q\ V2x)))))) \Leftrightarrow (\forall V3x \in \\ A_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ (p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))))) \Rightarrow \\ (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27) \\ V5y_27)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in \\ A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap \\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V2t1)\ V3t2) = V3t2)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0G \in ((ty_2Elist_2Elist\ A.27a)^{A.27a}).(\forall V1f \in (\\
& \quad (A.27b^{A.27b})^{A.27a}).(\forall V2seen \in (ty_2Elist_2Elist\ A.27a). \\
& \quad (\forall V3acc \in A.27b.(\forall V4visit_now \in A.27a.(\forall V5visit_later \in \\
& \quad (ty_2Elist_2Elist\ A.27a).((p\ (ap\ (c_2Epred_set_2EFINITE\ A.27a) \\
& \quad (ap\ (c_2EdirGraph_2EParents\ A.27a\ A.27a)\ V0G))) \Rightarrow (((ap\ (ap\ (ap \\
& \quad (ap\ (ap\ (c_2Edft_2EDFT\ A.27b\ A.27a)\ V0G)\ V1f)\ V2seen)\ (c_2Elist_2ENIL \\
& \quad A.27a))\ V3acc) = V3acc) \wedge ((ap\ (ap\ (ap\ (ap\ (ap\ (c_2Edft_2EDFT\ A.27b \\
& \quad A.27a)\ V0G)\ V1f)\ V2seen)\ (ap\ (ap\ (c_2Elist_2ECONS\ A.27a)\ V4visit_now) \\
& \quad V5visit_later))\ V3acc) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A.27b)\ (\\
& \quad ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V4visit_now)\ (ap\ (c_2Elist_2ELIST_TO_SET \\
& \quad A.27a)\ V2seen)))\ (ap\ (ap\ (ap\ (ap\ (ap\ (c_2Edft_2EDFT\ A.27b\ A.27a) \\
& \quad V0G)\ V1f)\ V2seen)\ V5visit_later)\ V3acc))\ (ap\ (ap\ (ap\ (ap\ (ap\ (c_2Edft_2EDFT \\
& \quad A.27b\ A.27a)\ V0G)\ V1f)\ (ap\ (ap\ (c_2Elist_2ECONS\ A.27a)\ V4visit_now) \\
& \quad V2seen))\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A.27a)\ (ap\ V0G\ V4visit_now)) \\
& \quad V5visit_later))\ (ap\ (ap\ V1f\ V4visit_now)\ V3acc))))))))) \\
& \hspace{15em} (29)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0P \in (((((2^{A.27b})(ty_2Elist_2Elist\ A.27a))(ty_2Elist_2Elist\ A.27a))^{(A.27b^{A.27b})^{A.27a}})((ty_2Elist_2Elist\ A.27a)^{A.27a}) \\
& \quad ((\forall V1G \in ((ty_2Elist_2Elist\ A.27a)^{A.27a}).(\forall V2f \in \\
& \quad ((A.27b^{A.27b})^{A.27a}).(\forall V3seen \in (ty_2Elist_2Elist\ A.27a). \\
& \quad (\forall V4visit_now \in A.27a.(\forall V5visit_later \in (ty_2Elist_2Elist \\
& \quad A.27a).(\forall V6acc \in A.27b.((p\ (ap\ (ap\ (ap\ (ap\ (ap\ V0P\ V1G)\ V2f) \\
& \quad V3seen)\ (c_2Elist_2ENIL\ A.27a))\ V6acc) \wedge (((p\ (ap\ (c_2Epred_set_2EFINITE \\
& \quad A.27a)\ (ap\ (c_2EdirGraph_2EParents\ A.27a\ A.27a)\ V1G))) \wedge (\neg(p\ (\\
& \quad ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V4visit_now)\ (ap\ (c_2Elist_2ELIST_TO_SET \\
& \quad A.27a)\ V3seen)))))) \Rightarrow (p\ (ap\ (ap\ (ap\ (ap\ (ap\ V0P\ V1G)\ V2f)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad A.27a)\ V4visit_now)\ V3seen))\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A.27a) \\
& \quad (ap\ V1G\ V4visit_now))\ V5visit_later))\ (ap\ (ap\ V2f\ V4visit_now) \\
& \quad V6acc)))) \wedge (((p\ (ap\ (c_2Epred_set_2EFINITE\ A.27a)\ (ap\ (c_2EdirGraph_2EParents \\
& \quad A.27a\ A.27a)\ V1G))) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V4visit_now) \\
& \quad (ap\ (c_2Elist_2ELIST_TO_SET\ A.27a)\ V3seen)))))) \Rightarrow (p\ (ap\ (ap\ (ap \\
& \quad (ap\ (ap\ V0P\ V1G)\ V2f)\ V3seen)\ V5visit_later)\ V6acc)))) \Rightarrow (p\ (ap\ (\\
& \quad ap\ (ap\ (ap\ (ap\ V0P\ V1G)\ V2f)\ V3seen)\ (ap\ (ap\ (c_2Elist_2ECONS\ A.27a) \\
& \quad V4visit_now)\ V5visit_later))\ V6acc)))))) \Rightarrow (\forall V7v \in \\
& \quad ((ty_2Elist_2Elist\ A.27a)^{A.27a}).(\forall V8v1 \in ((A.27b^{A.27b})^{A.27a}). \\
& \quad (\forall V9v2 \in (ty_2Elist_2Elist\ A.27a).(\forall V10v3 \in (ty_2Elist_2Elist \\
& \quad A.27a).(\forall V11v4 \in A.27b.(p\ (ap\ (ap\ (ap\ (ap\ (ap\ V0P\ V7v)\ V8v1) \\
& \quad V9v2)\ V10v3)\ V11v4))))))))) \\
& \hspace{15em} (30)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0G \in ((ty_2Elist_2Elist \\
& A_27a)^{A_27a}).(\forall V1f \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}). \\
& (\forall V2seen \in (ty_2Elist_2Elist\ A_27a).(\forall V3to_visit \in \\
& (ty_2Elist_2Elist\ A_27a).(\forall V4acc \in (ty_2Elist_2Elist \\
& A_27a).(\forall V5a \in (ty_2Elist_2Elist\ A_27a).(\forall V6b \in \\
& (ty_2Elist_2Elist\ A_27a).(((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a) \\
& (ap\ (c_2EdirGraph_2EParents\ A_27a\ A_27a)\ V0G))) \wedge ((V1f = (c_2Elist_2ECONS \\
& A_27a) \wedge (V4acc = (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V5a)\ V6b)))) \Rightarrow \\
& ((ap\ (ap\ (ap\ (ap\ (ap\ (c_2Edft_2EDFT\ (ty_2Elist_2Elist\ A_27a)\ A_27a) \\
& V0G)\ V1f)\ V2seen)\ V3to_visit)\ V4acc) = (ap\ (ap\ (c_2Elist_2EAPPEND \\
& A_27a)\ (ap\ (ap\ (ap\ (ap\ (ap\ (c_2Edft_2EDFT\ (ty_2Elist_2Elist\ A_27a) \\
& A_27a)\ V0G)\ V1f)\ V2seen)\ V3to_visit)\ V5a))\ V6b)))))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist \\
& A_27a).((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ (c_2Elist_2ENIL\ A_27a)) \\
& V0l) = V0l) \wedge (\forall V1l1 \in (ty_2Elist_2Elist\ A_27a).(\forall V2l2 \in \\
& (ty_2Elist_2Elist\ A_27a).(\forall V3h \in A_27a.((ap\ (ap\ (c_2Elist_2EAPPEND \\
& A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V3h)\ V1l1))\ V2l2) = (ap\ (ap \\
& (c_2Elist_2ECONS\ A_27a)\ V3h)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a) \\
& V1l1)\ V2l2))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0f \in ((A_27b^{A_27b})^{A_27a}).(\forall V1e \in A_27b.((ap\ (\\
& ap\ (ap\ (c_2Elist_2EFOLDR\ A_27a\ A_27b)\ V0f)\ V1e)\ (c_2Elist_2ENIL \\
& A_27a)) = V1e))) \wedge (\forall V2f \in ((A_27b^{A_27b})^{A_27a}).(\forall V3e \in \\
& A_27b.(\forall V4x \in A_27a.(\forall V5l \in (ty_2Elist_2Elist\ A_27a). \\
& ((ap\ (ap\ (ap\ (c_2Elist_2EFOLDR\ A_27a\ A_27b)\ V2f)\ V3e)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& A_27a)\ V4x)\ V5l)) = (ap\ (ap\ V2f\ V4x)\ (ap\ (ap\ (ap\ (c_2Elist_2EFOLDR \\
& A_27a\ A_27b)\ V2f)\ V3e)\ V5l))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\
& (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\
& A_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\
& c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\
& A_27a).(p\ (ap\ V0P\ V3l))))))
\end{aligned} \tag{34}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{35}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (39)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (40)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (41)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (42)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (43)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (49)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0G \in ((ty_2Elist_2Elist\ A_27a)^{A_27a}). (\forall V1f \in (\\ & \quad (A_27b^{A_27b})^{A_27a}). (\forall V2seen \in (ty_2Elist_2Elist\ A_27a). \\ & \quad (\forall V3to_visit \in (ty_2Elist_2Elist\ A_27a). (\forall V4acc \in \\ & \quad A_27b. ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ (ap\ (c_2EdirGraph_2EParents \\ & \quad A_27a\ A_27a)\ V0G))) \Rightarrow ((ap\ (ap\ (ap\ (ap\ (ap\ (c_2Edft_2EDFT\ A_27b\ A_27a) \\ & \quad V0G)\ V1f)\ V2seen)\ V3to_visit)\ V4acc) = (ap\ (ap\ (ap\ (c_2Elist_2EFOLDR \\ & \quad A_27a\ A_27b)\ V1f)\ V4acc)\ (ap\ (ap\ (ap\ (ap\ (ap\ (c_2Edft_2EDFT\ (ty_2Elist_2Elist \\ & \quad A_27a)\ A_27a)\ V0G)\ (c_2Elist_2ECONS\ A_27a))\ V2seen)\ V3to_visit) \\ & \quad (c_2Elist_2ENIL\ A_27a)))))))))) \end{aligned}$$