thm_2Edft_2EDFT__REACH__1 (TMaqfVJE3gWzwnZRdpFfr8cfo2SGNXpcjZ2)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^{2}))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ecombin_2EC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^{A_27a}$

Definition 6 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Definition 7 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Definition 8 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}$

Definition 9 *We define* c_2Ecombin_2EI *to be* $\lambda A_27a : \iota.(ap\ (ap\ ($c_2Ecombin_2ES$\ A_27a\ (A_27a^{A_27a})\ A_$

Definition 10 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 2$)\ (\lambda V0t \in 2.V0t))$.

Definition 11 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 12 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 2$)\ (\lambda V2t \in$

Definition 13 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (1)$$

Let $c\_2Edft\_2EDFT : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Edft\_2EDFT$$
$$A\_27a\ A\_27b \in (((((A\_27a^{A\_27a})^{(ty\_2Elist\_2Elist\ A\_27b)})^{(ty\_2Elist\_2Elist\ A\_27b)})^{((A\_27a^{A\_27a})^{A\_27b})})^{((ty\_2Elist\_2E$$
(2)

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{3}$$

Definition 14 *We define* $\mathsf{c\_2Ebool\_2E\_7E}$ *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{4}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{5}$$

Definition 15 *We define* $\mathsf{c\_2Epair\_2E\_2C}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{6}$$

Definition 16 *We define* $\mathsf{c\_2EdirGraph\_2EParents}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0G \in ((ty\_2Elist\_2Elist\ A$

Definition 17 *We define* $\mathsf{c\_2Ebool\_2EIN}$ *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 18 *We define* $\mathsf{c\_2Ebool\_2E\_5C\_2F}$ *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 19 *We define* $\mathsf{c\_2Epred\_\_set\_2EINSERT}$ *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 20 *We define* $\mathsf{c\_2Epred\_\_set\_2EEMPTY}$ *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2EF})$.

Definition 21 *We define* $\mathsf{c\_2Epred\_\_set\_2EFINITE}$ *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ (2$

Let $c\_2Elist\_2ELIST\_\_TO\_\_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a \in ((2^{A\_27a})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{7}$$

Definition 22 *We define* $\mathsf{c\_2Erelation\_2ERTC}$ *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2$

Definition 23 *We define* $\mathsf{c\_2EdirGraph\_2EREACH}$ *to be* $\lambda A\_27a : \iota.\lambda V0G \in ((ty\_2Elist\_2Elist\ A\_27a)^{A\_27a}$

Definition 24 *We define* $\mathsf{c\_2Ebool\_2E\_3F}$ *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 25 *We define* c_2EdirGraph_2EREACH__LIST *to be* $\lambda A\_27a:\iota.\lambda V0G \in ((ty\_2Elist\_2Elist\ A\_27a$

Let $c\_2Elist\_2EAPPEND : \iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{8}$$

Let $c\_2Elist\_2ECONS : \iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{9}$$

Assume the following.

$$True \tag{10}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{11}$$

Assume the following.

$$(\forall V0t \in 2.(False\Rightarrow(p\ V0t))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t)\vee(\neg(p\ V0t)))) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t))\Leftrightarrow(p\ V0t))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t \in 2.((\exists V1x \in A\_27a.(p\ V0t))\Leftrightarrow(p\ V0t))) \tag{15}$$

Assume the following.

$$(\forall V0t \in 2.(((True\wedge(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\wedge True)\Leftrightarrow(p\ V0t))\wedge(((False\wedge(p\ V0t))\Leftrightarrow False)\wedge((((p\ V0t)\wedge False)\Leftrightarrow False)\wedge(((p\ V0t)\wedge(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(((True\vee(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\vee True)\Leftrightarrow True)\wedge(((False\vee(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\vee False)\Leftrightarrow(p\ V0t))\wedge(((p\ V0t)\vee(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{18}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\forall V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A\_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.(((\forall V2x \in A\_27a.(p\ (ap\ V0P\ V2x))) \wedge (p\ V1Q)) \Leftrightarrow (\forall V3x \in A\_27a.((p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p\ V0P) \wedge (\forall V2x \in A\_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A\_27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x))))))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in A\_27a.(p\ (ap\ V1P\ V3x))) \vee (p\ V0Q))))) \tag{28}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.((((p\ V1B) \wedge (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A))))))) \tag{29}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{30}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.(((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27))))))))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a.(\forall V5y\_27 \in A\_27a.((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)\ V5y\_27))))))))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \tag{33}$$

Assume the following.

$$\begin{gathered}
\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
\forall V0G \in ((ty_2Elist_2Elist\ A_27a)^{A_27a}).(\forall V1f \in (\\
(A_27b^{A_27b})^{A_27a}).(\forall V2seen \in (ty_2Elist_2Elist\ A_27a). \\
(\forall V3acc \in A_27b.(\forall V4visit__now \in A_27a.(\forall V5visit__later \in \\
(ty_2Elist_2Elist\ A_27a).((p\ (ap\ (\mathsf{c_2Epred__set_2EFINITE}\ A_27a) \\
(ap\ (\mathsf{c_2EdirGraph_2EParents}\ A_27a\ A_27a)\ V0G))) \Rightarrow (((ap\ (ap\ (ap \\
(ap\ (ap\ (c_2Edft_2EDFT\ A_27b\ A_27a)\ V0G)\ V1f)\ V2seen)\ (c_2Elist_2ENIL \\
A_27a))\ V3acc) = V3acc) \wedge ((ap\ (ap\ (ap\ (ap\ (ap\ (c_2Edft_2EDFT\ A_27b \\
A_27a)\ V0G)\ V1f)\ V2seen)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V4visit__now) \\
V5visit__later))\ V3acc) = (ap\ (ap\ (ap\ (\mathsf{c_2Ebool_2ECOND}\ A_27b)\ (\\
ap\ (ap\ (\mathsf{c_2Ebool_2EIN}\ A_27a)\ V4visit__now)\ (ap\ (c_2Elist_2ELIST__TO__SET \\
A_27a)\ V2seen)))\ (ap\ (ap\ (ap\ (ap\ (ap\ (c_2Edft_2EDFT\ A_27b\ A_27a) \\
V0G)\ V1f)\ V2seen)\ V5visit__later)\ V3acc))\ (ap\ (ap\ (ap\ (ap\ (ap\ (c_2Edft_2EDFT \\
A_27b\ A_27a)\ V0G)\ V1f)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V4visit__now) \\
V2seen))\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ (ap\ V0G\ V4visit__now)) \\
V5visit__later))\ (ap\ (ap\ V1f\ V4visit__now)\ V3acc)))))))))))
\end{gathered} \tag{34}$$

Assume the following.

$$\begin{gathered}
\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
\forall V0P \in (((((2^{A_27b})^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)})^{((A_27b^{A_27b})^{A_27a})})^{((ty_2Elist_2Elist\ A_27a)^{A- \\
((\forall V1G \in ((ty_2Elist_2Elist\ A_27a)^{A_27a}).(\forall V2f \in \\
((A_27b^{A_27b})^{A_27a}).(\forall V3seen \in (ty_2Elist_2Elist\ A_27a). \\
(\forall V4visit__now \in A_27a.(\forall V5visit__later \in (ty_2Elist_2Elist \\
A_27a).(\forall V6acc \in A_27b.((p\ (ap\ (ap\ (ap\ (ap\ (ap\ V0P\ V1G)\ V2f) \\
V3seen)\ (c_2Elist_2ENIL\ A_27a))\ V6acc)) \wedge (((((p\ (ap\ (\mathsf{c_2Epred__set_2EFINITE} \\
A_27a)\ (ap\ (\mathsf{c_2EdirGraph_2EParents}\ A_27a\ A_27a)\ V1G))) \wedge (\neg(p\ (\\
ap\ (ap\ (\mathsf{c_2Ebool_2EIN}\ A_27a)\ V4visit__now)\ (ap\ (c_2Elist_2ELIST__TO__SET \\
A_27a)\ V3seen))))) \Rightarrow (p\ (ap\ (ap\ (ap\ (ap\ (ap\ V0P\ V1G)\ V2f)\ (ap\ (ap\ (c_2Elist_2ECONS \\
A_27a)\ V4visit__now)\ V3seen))\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a) \\
(ap\ V1G\ V4visit__now))\ V5visit__later))\ (ap\ (ap\ V2f\ V4visit__now) \\
V6acc)))) \wedge (((p\ (ap\ (\mathsf{c_2Epred__set_2EFINITE}\ A_27a)\ (ap\ (\mathsf{c_2EdirGraph_2EParents} \\
A_27a\ A_27a)\ V1G))) \wedge (p\ (ap\ (ap\ (\mathsf{c_2Ebool_2EIN}\ A_27a)\ V4visit__now) \\
(ap\ (c_2Elist_2ELIST__TO__SET\ A_27a)\ V3seen)))) \Rightarrow (p\ (ap\ (ap\ (ap \\
(ap\ (ap\ V0P\ V1G)\ V2f)\ V3seen)\ V5visit__later)\ V6acc)))) \Rightarrow (p\ (ap\ (\\
ap\ (ap\ (ap\ (ap\ V0P\ V1G)\ V2f)\ V3seen)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a) \\
V4visit__now)\ V5visit__later))\ V6acc)))))))))) \Rightarrow (\forall V7v \in \\
((ty_2Elist_2Elist\ A_27a)^{A_27a}).(\forall V8v1 \in ((A_27b^{A_27b})^{A_27a}). \\
(\forall V9v2 \in (ty_2Elist_2Elist\ A_27a).(\forall V10v3 \in (ty_2Elist_2Elist \\
A_27a).(\forall V11v4 \in A_27b.(p\ (ap\ (ap\ (ap\ (ap\ (ap\ V0P\ V7v)\ V8v1) \\
V9v2)\ V10v3)\ V11v4)))))))))
\end{gathered} \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0h \in A\_27b.(\forall V1t \in (ty\_2Elist\_2Elist\ A\_27b).(((ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a)) = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)) \wedge ((ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27b)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V0h)\ V1t)) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27b)\ V0h)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27b)\ V1t)))))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0e \in A\_27a.(\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l2 \in (ty\_2Elist\_2Elist\ A\_27a). ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0e)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1l1)\ V2l2)))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0e)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ V1l1))) \vee (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0e)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ V2l2)))))))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a)))) \Leftrightarrow False)) \wedge (\forall V1x \in A\_27a.(\forall V2h \in A\_27a.(\forall V3t \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1x)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V3t)))) \Leftrightarrow ((V1x = V2h) \vee (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1x)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ V3t)))))))))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(\forall V2s \in (2^{A\_27a}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V1y)\ V2s)\ V0x)) \Leftrightarrow ((V0x = V1y) \vee (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ V2s))))))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). ((\forall V1x \in A\_27a.(p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ V0R)\ V1x)\ V1x))) \wedge (\forall V2x \in A\_27a.(\forall V3y \in A\_27a.(\forall V4z \in A\_27a.(((p\ (ap\ (ap\ V0R\ V2x)\ V3y)) \wedge (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ V0R)\ V3y)\ V4z))) \Rightarrow (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ V0R)\ V2x)\ V4z)))))))) \tag{40}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{41}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg (p\ V0A)) \Rightarrow False))) \tag{42}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg ((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg (p\ V1B)) \Rightarrow False))))) \tag{43}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg ((\neg (p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg (p\ V1B)) \Rightarrow False))))) \tag{44}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg (p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{45}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg (p\ V2r)) \vee (\neg (p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg (p\ V2r)) \vee (\neg (p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg (p\ V1q)) \vee (\neg (p\ V0p))))))))))) \tag{46}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg (p\ V1q)) \vee (\neg (p\ V2r)))) \wedge (((p\ V1q) \vee (\neg (p\ V0p))) \wedge ((p\ V2r) \vee (\neg (p\ V0p))))))))) \tag{47}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg (p\ V1q))) \wedge (((p\ V0p) \vee (\neg (p\ V2r))) \wedge ((p\ V1q) \vee ((p\ V2r) \vee (\neg (p\ V0p)))))))))) \tag{48}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg (p\ V2r))) \wedge ((\neg (p\ V1q)) \vee ((p\ V2r) \vee (\neg (p\ V0p)))))))))) \tag{49}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p) \Leftrightarrow (\neg (p\ V1q))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge ((\neg (p\ V1q)) \vee (\neg (p\ V0p))))))) \tag{50}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg ((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (p\ V0p)))) \tag{51}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg ((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (\neg (p\ V1q))))) \tag{52}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0G \in ((ty\_2Elist\_2Elist\ A\_27a)^{A\_27a}).(\forall V1f \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}).(\forall V2seen \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V3to\_\_visit \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V4acc \in (ty\_2Elist\_2Elist\ A\_27a).(((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (ap\ (\mathsf{c\_2EdirGraph\_2EParents}\ A\_27a\ A\_27a)\ V0G))) \wedge (V1f = (c\_2Elist\_2ECONS\ A\_27a))) \Rightarrow (\forall V5x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V5x)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Edft\_2EDFT\ (ty\_2Elist\_2Elist\ A\_27a)\ A\_27a)\ V0G)\ V1f)\ V2seen)\ V3to\_\_visit)\ V4acc)))) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V5x)\ (ap\ (ap\ (\mathsf{c\_2EdirGraph\_2EREACH\_\_LIST}\ A\_27a)\ V0G)\ V3to\_\_visit))) \vee (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V5x)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ V4acc)))))))))))$$