

thm_2EdirGraph_2EREACH_EXCLUDE
(TMW-
eRyp8owhrcm86Kq3CHvpBWY4QRxaubmH)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 8 We define $c_2ERelation_2ERTC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1a \in A_27a.\lambda V2b \in A_27a.$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2ELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ELIST_TO_SET A_27a \in ((2^{A_27a})^{(ty_2Elist_2Elist A_27a)}) \quad (2)$$

Definition 9 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 10 We define $c_2EdirGraph_2EREACH$ to be $\lambda A_27a : \iota.\lambda V0G \in ((ty_2Elist_2Elist A_27a)^{A_27a})$

Definition 11 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 12 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($
Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (3)$$

Definition 13 We define $c_2EdirGraph_2EEXCLUDE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0G \in ((ty_2Elist_2Elist$

Definition 14 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$
Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (4)$$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (6)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (7)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \vee (\neg(p\ V0t)))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (9)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (10)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (11)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27b^{A_27a}). ((V0f = V1g) \Leftrightarrow (\forall V2x \in A_27a. ((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \quad (12)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p V0t))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0t1 \in A_{.27a}.(\forall V1t2 \in \\
& A_{.27a}.(((ap (ap (ap (c_{.2Ebool_2ECOND} A_{.27a}) c_{.2Ebool_2ET}) V0t1) \\
& V1t2) = V0t1) \wedge ((ap (ap (ap (c_{.2Ebool_2ECOND} A_{.27a}) c_{.2Ebool_2EF} \\
& V0t1) V1t2) = V1t2))))))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty A_{.27a} \Rightarrow ((\forall V0x \in A_{.27a}.((p (ap (ap \\
& (c_{.2Ebool_2EIN} A_{.27a}) V0x) (ap (c_{.2Elist_2ELIST_TO_SET} A_{.27a}) \\
& (c_{.2Elist_2ENIL} A_{.27a})))) \Leftrightarrow False)) \wedge (\forall V1x \in A_{.27a}.(\forall V2h \in \\
& A_{.27a}.(\forall V3t \in (ty_{.2Elist_2Elist} A_{.27a}).((p (ap (ap (c_{.2Ebool_2EIN} \\
& A_{.27a}) V1x) (ap (c_{.2Elist_2ELIST_TO_SET} A_{.27a}) (ap (ap (c_{.2Elist_2ECONS} \\
& A_{.27a}) V2h) V3t)))) \Leftrightarrow ((V1x = V2h) \vee (p (ap (ap (c_{.2Ebool_2EIN} A_{.27a}) \\
& V1x) (ap (c_{.2Elist_2ELIST_TO_SET} A_{.27a}) V3t))))))))))
\end{aligned} \tag{15}$$

Theorem 1

$$\begin{aligned}
& \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0G \in ((ty_{.2Elist_2Elist} \\
& A_{.27a})^{A_{.27a}}).(\forall V1x \in (2^{A_{.27a}}).((ap (c_{.2EdirGraph_2EREACH} \\
& A_{.27a}) (ap (ap (c_{.2EdirGraph_2EEXCLUDE} A_{.27a} A_{.27a}) V0G) V1x)) = \\
& (ap (c_{.2Erelation_2ERTC} A_{.27a}) (\lambda V2x_{.27} \in A_{.27a}.(\lambda V3y \in \\
& A_{.27a}.(ap (ap c_{.2Ebool_2E_2F_5C} (ap c_{.2Ebool_2E_7E} (ap (ap (c_{.2Ebool_2EIN} \\
& A_{.27a}) V2x_{.27}) V1x))) (ap (ap (c_{.2Ebool_2EIN} A_{.27a}) V3y) (ap (c_{.2Elist_2ELIST_TO_SET} \\
& A_{.27a}) (ap V0G V2x_{.27}))))))))))
\end{aligned}$$