

# thm\_2Edivides\_2EDIVIDES\_\_LEQ\_\_OR\_\_ZERO (TMQUwC4v194AriE2FX5gCpjadZy5mycEVns)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (2)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (3)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (4)$$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num (c\_2Enum\_2ESUC\_REP m))$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_Emin\_2E\_40$

**Definition 11** We define  $c\_Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 12** We define  $c\_Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 13** We define  $c\_Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (5)$$

**Definition 14** We define  $c\_Edivides\_2Edivides$  to be  $\lambda V0a \in ty\_2Enum\_2Enum. \lambda V1b \in ty\_2Enum\_2Enum$

Let  $c\_Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_Enum\_2EZERO\_REP \in \omega \quad (6)$$

**Definition 15** We define  $c\_Enum\_2E0$  to be  $(ap\ c\_Enum\_2EABS\_num\ c\_Enum\_2EZERO\_REP)$ .

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. ((V0m = c\_Enum\_2E0) \vee (\exists V1n \in ty\_2Enum\_2Enum. (V0m = (ap\ c\_Enum\_2ESUC\ V1n)))))) \quad (7)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_Earithmetic\_2E\_2A\ V0m)\ V1n) = (ap\ (ap\ c\_Earithmetic\_2E\_2A\ V1n)\ V0m)))) \quad (8)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ((ap\ (ap\ c\_Earithmetic\_2E\_2A\ V0m)\ V1n) = c\_Enum\_2E0) \Leftrightarrow ((V0m = c\_Enum\_2E0) \vee (V1n = c\_Enum\_2E0)))) \quad (9)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ((p\ (ap\ (ap\ c\_Earithmetic\_2E\_3C\_3D\ V0m)\ (ap\ (ap\ c\_Earithmetic\_2E\_2A\ V0m)\ V1n))) \Leftrightarrow ((V0m = c\_Enum\_2E0) \vee (p\ (ap\ (ap\ c\_Eprim\_rec\_2E\_3C\ c\_Enum\_2E0)\ V1n)))) \wedge ((p\ (ap\ (ap\ c\_Earithmetic\_2E\_3C\_3D\ V0m)\ (ap\ (ap\ c\_Earithmetic\_2E\_2A\ V1n)\ V0m))) \Leftrightarrow ((V0m = c\_Enum\_2E0) \vee (p\ (ap\ (ap\ c\_Eprim\_rec\_2E\_3C\ c\_Enum\_2E0)\ V1n)))))) \quad (10)$$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p \ V0t))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (14)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (15)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2) \Rightarrow (p \ V2t3)) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (18)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.(((p \ V0x) \Leftrightarrow (p \ V1x\_27)) \wedge ((p \ V1x\_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y\_27)))) \Rightarrow (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x\_27) \Rightarrow (p \ V3y\_27)))))) \quad (19)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg((ap \ c\_2Enum\_2ESUC \ V0n) = c\_2Enum\_2E0))) \quad (20)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg(p \ (ap \ (ap \ c\_2Eprim\_rec\_2E\_3C \ V0n) \ c\_2Enum\_2E0)))) \quad (21)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p \ (ap \ (ap \ c\_2Eprim\_rec\_2E\_3C \ c\_2Enum\_2E0) \ (ap \ c\_2Enum\_2ESUC \ V0n)))) \quad (22)$$

**Theorem 1**

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Edivides\_2Edivides V0m) V1n)) \Rightarrow ((p (ap (ap c\_2Arithmetic\_2E\_3C\_3D V0m) V1n)) \vee (V1n = c\_2Enum\_2E0))))))$$