

# thm\_2Edivides\_2EPRIME\_3

(TMQ3i4K2kBWt9zEm9NM2giQogNhXLUbWz3S)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (2)$$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (3)$$

**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A . \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap V0P (ap (c\_2Emin\_2E\_40 A$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (4)$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})$

**Definition 6** We define  $c\_2Edivides\_2Edivides$  to be  $\lambda V0a \in ty\_2Enum\_2Enum. \lambda V1b \in ty\_2Enum\_2Enum. ap (c\_2Edivides_2Edivides (V0a, V1b))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in omega \quad (5)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (6)$$

**Definition 7** We define  $c\_2Enum\_2E0$  to be (*ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP*).

**Definition 8** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREPE\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{t y\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^\omega)^\omega \quad (8)$$

**Definition 9** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ 0)$

Let  $c_2Earithmetic_2E_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (9)$$

**Definition 10** We define  $c\_2Earthmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earthmetic\ 1\ n)\ V)$

**Definition 11** We define  $c\_2Earthmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x.$

**Definition 12** We define  $c_2 \in \text{Emin\_2E\_3D\_3D\_3E}$  to be  $\lambda P \in 2.\lambda Q \in 2.\text{inj\_o} (p \ P \Rightarrow p \ Q)$  of type  $\iota$ .

**Definition 13** We define  $c\_Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_2E\_21 2) (\lambda V2t \in$

**Definition 14** We define  $c\_2Ebool\_2E\text{EF}$  to be  $(ap\ (c\_2Ebool\_2E\text{21}\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 15** We define  $c_{\text{Ebool}} \cdot 2\text{E} \cdot 7\text{E}$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c_{\text{Emin}} \cdot 2\text{E} \cdot 3\text{D} \cdot 3\text{D} \cdot 3\text{E}\ V0t)\ c_{\text{Ebool}} \cdot 2\text{E})$

**Definition 16** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 17** We define  $c_2\text{Edivides}_2\text{Eprime}$  to be  $\lambda V0a \in ty\_2Enum\_2Enum.(ap (ap (c_2\text{Ebool}_2E\_2F\_5C))$

**Definition 18** We define  $c_2Eprim\_rec_2E_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 19** We define  $\in$  to be arithmetic if there exists  $\lambda V0m \in tu$  such that  $\lambda V1n \in tu$  and  $\lambda V0m \in tu$  and  $\lambda V1n \in tu$ .

**Definition 20.** We define  $c$  2Earthmetic 2E 3E 3D to be  $\lambda V0m \in tu\ 2Enum\ 2Enum\ \lambda V1n \in tu\ 2Enum\ 2Enum\ \lambda V2o \in tu\ 2Enum\ 2Enum\ \lambda V3p \in tu\ 2Enum\ 2Enum\ \lambda V4q \in tu\ 2Enum\ 2Enum\ \lambda V5r \in tu\ 2Enum\ 2Enum\ \lambda V6s \in tu\ 2Enum\ 2Enum\ \lambda V7t \in tu\ 2Enum\ 2Enum\ \lambda V8u \in tu\ 2Enum\ 2Enum\ \lambda V9v \in tu\ 2Enum\ 2Enum\ \lambda V10w \in tu\ 2Enum\ 2Enum\ \lambda V11x \in tu\ 2Enum\ 2Enum\ \lambda V12y \in tu\ 2Enum\ 2Enum\ \lambda V13z \in tu\ 2Enum\ 2Enum$

Let  $c$  be a arithmetic  $\text{FEXP}$ ; be given. Assume the following

$$c\_2Earithmetic\_2EXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (10)$$

Let  $c_2 \in \text{arithmetic}_2E_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \\ (11)$$

**Definition 21** We define  $c\_2EiSUC$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2ESUC\ (ap\ c\_2EiSUC\ V0n))$

**Definition 22** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ c\_2Earithmetic\_2E\_3C\_3D\ V0m\ V1n)$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (12)$$

Let  $c\_2Enumeral\_2Eonecount : \iota$  be given. Assume the following.

$$c\_2Enumeral\_2Eonecount \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (13)$$

Let  $c\_2Enumeral\_2Eexactlog : \iota$  be given. Assume the following.

$$c\_2Enumeral\_2Eexactlog \in (ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum} \quad (14)$$

**Definition 23** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap\ c\_2Ebool\_2ECOND\ V0t\ V1t1\ V2t2))))$

**Definition 24** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ V0m))))$

**Definition 25** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ (c\_2Earithmetic\_2EEVEN\ V0n)))$

**Definition 26** We define  $c\_2Earithmetic\_2EDIV2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ (c\_2Earithmetic\_2EEVEN\ V0n)))$

Let  $c\_2Enumeral\_2Etexp\_help : \iota$  be given. Assume the following.

$$c\_2Enumeral\_2Etexp\_help \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (15)$$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (16)$$

**Definition 27** We define  $c\_2Ebool\_2ELET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27a.(ap\ c\_2Ebool\_2ELET\ V0f\ V1x)))$

**Definition 28** We define  $c\_2Enumeral\_2EiDUB$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2EEVEN\ V0x)))$

**Definition 29** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 30** We define  $c\_2Enumeral\_2Einternal\_mult$  to be  $c\_2Earithmetic\_2E\_2A$ .

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ c\_2Enum\_2E0)\ V0n))) \quad (17)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge \\
& ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap \\
& (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) \\
& V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A \\
& V0m) V1n)))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2A \\
& V1n) V0m)))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))) \Rightarrow (p ( \\
& ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V2p))))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) (ap (ap c\_2Earithmetic\_2E\_2B \\
& V0m) V2p))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
& V0m)) V1n)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
& V1n)) V0m))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. ((ap c\_2Enum\_2ESUC V0n) = (ap (ap \\
& c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& c\_2Earithmetic\_2EZERO))) V0n)))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1k \in ty\_2Enum\_2Enum. ( \\
 & \quad \forall V2q \in ty\_2Enum\_2Enum. ((\exists V3r \in ty\_2Enum\_2Enum. ( \\
 & (V1k = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A \\
 & \quad V2q) V0n)) V3r)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C V3r) V0n)))) \Rightarrow ( \\
 & \quad (ap (ap c_2Earithmetic_2EDIV V1k) V0n) = V2q))))))) \\
 & \quad (24)
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1k \in ty\_2Enum\_2Enum. ( \\
 & \quad \forall V2r \in ty\_2Enum\_2Enum. ((\exists V3q \in ty\_2Enum\_2Enum. ( \\
 & (V1k = (ap (ap c_2Earithmetic_2E_2B (ap (ap (ap c_2Earithmetic_2E_2A \\
 & \quad V3q) V0n)) V2r)) \wedge (p (ap (ap (ap c_2Eprim_rec_2E_3C V2r) V0n)))) \Rightarrow ( \\
 & \quad (ap (ap c_2Earithmetic_2EMOD V1k) V0n) = V2r))))))) \\
 \end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty\_2Enum\_2Enum. (\forall V1y \in ty\_2Enum\_2Enum. (\forall V2z \in ty\_2Enum\_2Enum. (((ap (ap c\_2Earithmetic\_2E\_2A \\
 & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 V0x))) \\
 & V1y) = (ap c\_2Earithmetic\_2ENUMERAL V2z)) \Leftrightarrow ((V1y = (ap (ap c\_2Earithmetic\_2EDIV \\
 & (ap c\_2Earithmetic\_2ENUMERAL V2z)) (ap c\_2Earithmetic\_2ENUMERAL \\
 & (ap c\_2Earithmetic\_2EBIT1 V0x)))) \wedge ((ap (ap c\_2Earithmetic\_2EMOD \\
 & (ap c\_2Earithmetic\_2ENUMERAL V2z)) (ap c\_2Earithmetic\_2ENUMERAL \\
 & (ap c\_2Earithmetic\_2EBIT1 V0x))) = c\_2Enum\_2E0))) \wedge (((ap (ap c\_2Earithmetic\_2E\_2A \\
 & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 V0x))) \\
 & V1y) = (ap c\_2Earithmetic\_2ENUMERAL V2z)) \Leftrightarrow ((V1y = (ap (ap c\_2Earithmetic\_2EDIV \\
 & (ap c\_2Earithmetic\_2ENUMERAL V2z)) (ap c\_2Earithmetic\_2ENUMERAL \\
 & (ap c\_2Earithmetic\_2EBIT2 V0x)))) \wedge ((ap (ap c\_2Earithmetic\_2EMOD \\
 & (ap c\_2Earithmetic\_2ENUMERAL V2z)) (ap c\_2Earithmetic\_2ENUMERAL \\
 & (ap c\_2Earithmetic\_2EBIT2 V0x))) = c\_2Enum\_2E0)))))))
 \end{aligned} \tag{26}$$

Assume the following.

*True* (27)

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2))))))) \quad (28)$$

Assume the following.

$$(\forall V \exists t \in 2. (False \Rightarrow (p \ V \ 0 \ t))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (30)$$

Assume the following.

$$\forall A_{\text{27a}}.\text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_{\text{27a}}.(p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (31)$$

Assume the following.

$$\forall A_{\text{27a}}.\text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0t \in 2.((\exists V1x \in A_{\text{27a}}.(p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (32)$$

Assume the following.

$$(\forall V0t \in 2.(((\text{True} \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge \text{True}) \Leftrightarrow (p \ V0t)) \wedge (((\text{False} \wedge (p \ V0t)) \Leftrightarrow \text{False}) \wedge (((p \ V0t) \wedge \text{False}) \Leftrightarrow \text{False}) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2.(((\text{True} \vee (p \ V0t)) \Leftrightarrow \text{True}) \wedge (((p \ V0t) \vee \text{True}) \Leftrightarrow \text{True}) \wedge (((\text{False} \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \text{False}) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (34)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t)) \Leftrightarrow (p \ V0t))) \wedge (((\neg \text{True}) \Leftrightarrow \text{False}) \wedge ((\neg \text{False}) \Leftrightarrow \text{True}))) \quad (35)$$

Assume the following.

$$\forall A_{\text{27a}}.\text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0x \in A_{\text{27a}}.((V0x = V0x) \Leftrightarrow \text{True})) \quad (36)$$

Assume the following.

$$\forall A_{\text{27a}}.\text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0x \in A_{\text{27a}}.(\forall V1y \in A_{\text{27a}}.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (37)$$

Assume the following.

$$(\forall V0t \in 2.(((\text{True} \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow \text{True}) \Leftrightarrow (p \ V0t)) \wedge (((\text{False} \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow \text{False}) \Leftrightarrow (\neg(p \ V0t)))))) \quad (38)$$

Assume the following.

$$\begin{aligned} \forall A_{\text{27a}}.\text{nonempty } A_{\text{27a}} \Rightarrow & (\forall V0t1 \in A_{\text{27a}}.(\forall V1t2 \in A_{\text{27a}}.((ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A_{\text{27a}}) \ c\_2Ebool\_2ET) \ V0t1) \\ & V1t2) = V0t1) \wedge ((ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A_{\text{27a}}) \ c\_2Ebool\_2EF) \ V0t1) \ V1t2) = V1t2)))))) \end{aligned} \quad (39)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B)) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))) \quad (42)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (43)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (44)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Edivides_2Edivides c_2Enum_2E0) V0m)) \Leftrightarrow (V0m = c_2Enum_2E0))) \quad (45)$$

Assume the following.

$$(\forall V0a \in ty\_2Enum\_2Enum. (\forall V1b \in ty\_2Enum\_2Enum. (((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V1b)) \wedge (p (ap (ap c_2Edivides_2Edivides V0a) V1b))) \Rightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V0a) V1b)))))) \quad (46)$$

Assume the following.

$$\begin{aligned} (((ap c_2Enum_2ESUC c_2Earithmetic_2EZERO) = (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)) \wedge ((\forall V0n \in ty\_2Enum\_2Enum. ((ap c_2Enum_2ESUC (ap c_2Earithmetic_2EBIT1 V0n)) = (ap c_2Earithmetic_2EBIT2 V0n))) \wedge (\forall V1n \in ty\_2Enum\_2Enum. ((ap c_2Enum_2ESUC (ap c_2Earithmetic_2EBIT2 V1n)) = (ap c_2Earithmetic_2EBIT1 (ap c_2Enum_2ESUC V1n))))))) \end{aligned} \quad (47)$$

Assume the following.

$((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in ty\_2Enum\_2Enum.((\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enum\_2EiZ (ap (ap c\_2Earithmetic\_2E\_2B V2n) V3m))))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum.((\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A V6n) V7m))))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in ty\_2Enum\_2Enum.((\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D V10n) V11m))))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEEXP V14n) c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge ((\forall V15n \in ty\_2Enum\_2Enum.((\forall V16m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2EEEXP V15n) V16m))))))) \wedge (((ap c\_2Enum\_2ESUC c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum.((ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enum\_2ESUC V17n))))))) \wedge (((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Eprim\_rec\_2EPRE V18n))))))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum.((\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m))))))) \wedge ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL V23n)) \Leftrightarrow False))) \wedge ((\forall V24n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL V24n)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.((\forall V26m \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL V25n) V26m))))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E c\_2Enum\_2E0) V27n)) \Leftrightarrow False))) \wedge ((\forall V28n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL V28n)) c\_2Enum\_2E0) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.((\forall V30m \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E c\_2Enum\_2E0) V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) V29n))))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3D c\_2Enum\_2E0) V31n)) \Leftrightarrow True))) \wedge ((\forall V32n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C c\_2Enum\_2E0) V32n)) \Leftrightarrow False))) \wedge ((\forall V33n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3D c\_2Enum\_2E0) V33n)) \Leftrightarrow True))) \wedge ((\forall V34n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3D c\_2Enum\_2E0) V34n)) \Leftrightarrow False)))$

Assume the following.

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
 & ((c\_2Earthmetic\_2EZERO = (ap\ c\_2Earthmetic\_2EBIT1\ V0n)) \Leftrightarrow False) \wedge \\
 & (((ap\ c\_2Earthmetic\_2EBIT1\ V0n) = c\_2Earthmetic\_2EZERO) \Leftrightarrow \\
 & False) \wedge ((c\_2Earthmetic\_2EZERO = (ap\ c\_2Earthmetic\_2EBIT2 \\
 & V0n)) \Leftrightarrow False) \wedge (((ap\ c\_2Earthmetic\_2EBIT2\ V0n) = c\_2Earthmetic\_2EZERO) \Leftrightarrow \\
 & False) \wedge (((ap\ c\_2Earthmetic\_2EBIT1\ V0n) = (ap\ c\_2Earthmetic\_2EBIT2 \\
 & V1m)) \Leftrightarrow False) \wedge (((ap\ c\_2Earthmetic\_2EBIT2\ V0n) = (ap\ c\_2Earthmetic\_2EBIT1 \\
 & V1m)) \Leftrightarrow False) \wedge (((ap\ c\_2Earthmetic\_2EBIT1\ V0n) = (ap\ c\_2Earthmetic\_2EBIT1 \\
 & V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ c\_2Earthmetic\_2EBIT2\ V0n) = (ap\ c\_2Earthmetic\_2EBIT2 \\
 & V1m)) \Leftrightarrow (V0n = V1m))))))) \\
 \end{aligned}$$

Assume the following.

Assume the following.

$(\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Earithmetic\_2EZERO) V0n)) \Leftrightarrow True) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1m) V0n)))) \wedge ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0n) V1m))))))))))))))$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (((ap c\_2Enumeral\_2EiDUB (ap c\_2Earithmetic\_2EBIT1 V0n)) = (ap c\_2Earithmetic\_2EBIT2 (ap c\_2Enumeral\_2EiDUB V0n))) \wedge (((ap c\_2Enumeral\_2EiDUB (ap c\_2Earithmetic\_2EBIT2 V0n)) = (ap c\_2Earithmetic\_2EBIT2 (ap c\_2Earithmetic\_2EBIT1 V0n)))) \wedge ((ap c\_2Enumeral\_2EiDUB c\_2Earithmetic\_2EZERO) = c\_2Earithmetic\_2EZERO))) \quad (53)$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. ((p (ap c\_2Earithmetic\_2EEVEN c\_2Earithmetic\_2EZERO))) \wedge \\
& \quad ((p (ap c\_2Earithmetic\_2EEVEN (ap c\_2Earithmetic\_2EBIT2 V0n)))) \wedge \\
& \quad ((\neg(p (ap c\_2Earithmetic\_2EEVEN (ap c\_2Earithmetic\_2EBIT1 V0n)))) \wedge \\
& \quad \quad ((\neg(p (ap c\_2Earithmetic\_2EODD c\_2Earithmetic\_2EZERO))) \wedge (( \\
& \quad \quad \neg(p (ap c\_2Earithmetic\_2EODD (ap c\_2Earithmetic\_2EBIT2 V0n)))) \wedge \\
& \quad \quad (p (ap c\_2Earithmetic\_2EODD (ap c\_2Earithmetic\_2EBIT1 V0n)))))))))) \\
& \hspace{10em} (54)
\end{aligned}$$

Assume the following.

$((\forall V0x \in ty\_2Enum\_2Enum.((ap (ap c\_2Enumeral\_2Eonecount c\_2Earithmetic\_2EZERO) V0x) = V0x)) \wedge (\forall V1n \in ty\_2Enum\_2Enum.(\forall V2x \in ty\_2Enum\_2Enum.((ap (ap c\_2Enumeral\_2Eonecount (ap c\_2Earithmetic\_2EBIT1 V1n)) V2x) = (ap (ap c\_2Enumeral\_2Eonecount V1n) (ap c\_2Enum\_2ESUC V2x)))))) \wedge (\forall V3n \in ty\_2Enum\_2Enum.(\forall V4x \in ty\_2Enum\_2Enum.((ap (ap c\_2Enumeral\_2Eonecount (ap c\_2Earithmetic\_2EBIT2 V3n)) V4x) = c\_2Earithmetic\_2EZERO))))))$  (55)

Assume the following.

$((ap\ c\_2E\!n\ u\!m\ e\!r\ a\!l\ _2E\!x\!a\!c\!t\ l\!o\ g\ c\_2E\!a\!r\!i\!t\ h\!e\!t\ i\!c\ _2E\!Z\!E\!R\!O)) = c\_2E\!a\!r\!i\!t\ h\!e\!t\ i\!c\ _2E\!Z\!E\!R\!O) \wedge$   
 $((\forall V0n \in ty\_2E\!n\ u\!m\ _2E\!n\ u\!m. ((ap\ c\_2E\!n\ u\!m\ e\!r\ a\!l\ _2E\!x\!a\!c\!t\ l\!o\ g\ ($   
 $ap\ c\_2E\!a\!r\!i\!t\ h\!e\!t\ i\!c\ _2E\!B\!I\!T\!1\ V0n)) = c\_2E\!a\!r\!i\!t\ h\!e\!t\ i\!c\ _2E\!Z\!E\!R\!O)) \wedge (\forall V1n \in$   
 $ty\_2E\!n\ u\!m\ _2E\!n\ u\!m. ((ap\ c\_2E\!n\ u\!m\ e\!r\ a\!l\ _2E\!x\!a\!c\!t\ l\!o\ g\ (ap\ c\_2E\!a\!r\!i\!t\ h\!e\!t\ i\!c\ _2E\!B\!I\!T\!2$   
 $V1n)) = (ap\ (ap\ (c\_2E\!b\!o\!o\!l\ _2E\!L\!E\!T\ ty\_2E\!n\ u\!m\ _2E\!n\ u\!m\ ty\_2E\!n\ u\!m\ _2E\!n\ u\!m\ )$   
 $(\lambda V2x \in ty\_2E\!n\ u\!m\ _2E\!n\ u\!m. (ap\ (ap\ (ap\ (c\_2E\!b\!o\!o\!l\ _2E\!C\!O\!N\!D\ ty\_2E\!n\ u\!m\ _2E\!n\ u\!m\ )$   
 $(ap\ (ap\ (c\_2E\!m\ i\!n\ _2E\!_3D\ ty\_2E\!n\ u\!m\ _2E\!n\ u\!m\ ) V2x) c\_2E\!a\!r\!i\!t\ h\!e\!t\ i\!c\ _2E\!Z\!E\!R\!O))$   
 $c\_2E\!a\!r\!i\!t\ h\!e\!t\ i\!c\ _2E\!Z\!E\!R\!O)\ (ap\ c\_2E\!a\!r\!i\!t\ h\!e\!t\ i\!c\ _2E\!B\!I\!T\!1\ V2x))))\ (ap\$   
 $(ap\ c\_2E\!n\ u\!m\ e\!r\ a\!l\ _2E\!o\!n\ e\!c\!o\!u\!n\ t\ V1n)\ c\_2E\!a\!r\!i\!t\ h\!e\!t\ i\!c\ _2E\!Z\!E\!R\!O)))))))$   
(56)

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1x \in ty\_2Enum\_2Enum. (\forall V2y \in ty\_2Enum\_2Enum. (((ap (ap c\_2Earithmetic\_2E\_2A c\_2Earithmetic\_2EZERO) \\
V0n) = c\_2Earithmetic\_2EZERO) \wedge (((ap (ap c\_2Earithmetic\_2E\_2A \\
V0n) c\_2Earithmetic\_2EZERO) = c\_2Earithmetic\_2EZERO) \wedge (((ap \\
(ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2EBIT1 V1x)) (ap \\
c\_2Earithmetic\_2EBIT1 V2y)) = (ap (ap c\_2Enumeral\_2Einternal\_mult \\
(ap c\_2Earithmetic\_2EBIT1 V1x)) (ap c\_2Earithmetic\_2EBIT1 V2y))) \wedge \\
(((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2EBIT1 V1x)) \\
(ap c\_2Earithmetic\_2EBIT2 V2y)) = (ap (ap (c\_2Ebool\_2ELET ty\_2Enum\_2Enum \\
ty\_2Enum\_2Enum) (\lambda V3n \in ty\_2Enum\_2Enum. (ap (ap (c\_2Ebool\_2ECOND \\
ty\_2Enum\_2Enum) (ap c\_2Earithmetic\_2EODD V3n)) (ap (ap c\_2Enumeral\_2Eexp\_help \\
(ap c\_2Earithmetic\_2EDIV2 V3n)) (ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmetic\_2EBIT1 \\
V1x)))) (ap (ap c\_2Enumeral\_2Einternal\_mult (ap c\_2Earithmetic\_2EBIT1 \\
V1x)) (ap c\_2Earithmetic\_2EBIT2 V2y)))))) (ap c\_2Enumeral\_2Eexactlog \\
(ap c\_2Earithmetic\_2EBIT2 V2y)))) \wedge (((ap (ap c\_2Earithmetic\_2E\_2A \\
(ap c\_2Earithmetic\_2EBIT2 V1x)) (ap c\_2Earithmetic\_2EBIT1 V2y)) = \\
(ap (ap (c\_2Ebool\_2ELET ty\_2Enum\_2Enum ty\_2Enum\_2Enum) (\lambda V4m \in \\
ty\_2Enum\_2Enum. (ap (ap (c\_2Ebool\_2ECOND ty\_2Enum\_2Enum) \\
(ap c\_2Earithmetic\_2EODD V4m)) (ap (ap c\_2Enumeral\_2Eexp\_help \\
(ap c\_2Earithmetic\_2EDIV2 V4m)) (ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmetic\_2EBIT1 \\
V2y)))))) (ap (ap c\_2Enumeral\_2Einternal\_mult (ap c\_2Earithmetic\_2EBIT2 \\
V1x)) (ap c\_2Earithmetic\_2EBIT1 V2y)))))) (ap c\_2Enumeral\_2Eexactlog \\
(ap c\_2Earithmetic\_2EBIT2 V1x)))) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A \\
(ap c\_2Earithmetic\_2EBIT2 V1x)) (ap c\_2Earithmetic\_2EBIT2 V2y)) = \\
(ap (ap (c\_2Ebool\_2ELET ty\_2Enum\_2Enum ty\_2Enum\_2Enum) (\lambda V5m \in \\
ty\_2Enum\_2Enum. (ap (ap (c\_2Ebool\_2ELET ty\_2Enum\_2Enum ty\_2Enum\_2Enum) \\
(\lambda V6n \in ty\_2Enum\_2Enum. (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Enum\_2Enum) \\
(ap c\_2Earithmetic\_2EODD V5m)) (ap (ap c\_2Enumeral\_2Eexp\_help \\
(ap c\_2Earithmetic\_2EDIV2 V5m)) (ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmetic\_2EBIT2 \\
V2y)))))) (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Enum\_2Enum) (ap c\_2Earithmetic\_2EODD \\
V6n)) (ap (ap c\_2Enumeral\_2Eexp\_help (ap c\_2Earithmetic\_2EDIV2 \\
V6n)) (ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmetic\_2EBIT2 V1x)))) \\
(ap (ap c\_2Enumeral\_2Einternal\_mult (ap c\_2Earithmetic\_2EBIT2 \\
V1x)) (ap c\_2Earithmetic\_2EBIT2 V2y)))))) (ap c\_2Enumeral\_2Eexactlog \\
(ap c\_2Earithmetic\_2EBIT2 V2y)))))) (ap c\_2Enumeral\_2Eexactlog \\
(ap c\_2Earithmetic\_2EBIT2 V1x)))))))))))))) \\
(57)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2EEnumeral\_2Einternal\_mult c\_2Earithmetic\_2EZERO) \\
& V0n) = c\_2Earithmetic\_2EZERO) \wedge (((ap (ap c\_2EEnumeral\_2Einternal\_mult \\
& V0n) c\_2Earithmetic\_2EZERO) = c\_2Earithmetic\_2EZERO) \wedge (((ap \\
& (ap c\_2EEnumeral\_2Einternal\_mult (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) V1m) = (ap c\_2EEnumeral\_2EiZ (ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap c\_2EEnumeral\_2EiDUB (ap (ap c\_2EEnumeral\_2Einternal\_mult \\
& V0n) V1m))) V1m))) \wedge ((ap (ap c\_2EEnumeral\_2Einternal\_mult (ap \\
& c\_2Earithmetic\_2EBIT2 V0n)) V1m) = (ap c\_2EEnumeral\_2EiDUB (ap \\
& c\_2EEnumeral\_2EiZ (ap (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2EEnumeral\_2Einternal\_mult \\
& V0n) V1m))))))) \\
\end{aligned} \tag{58}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{59}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \\
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \\
\end{aligned} \tag{62}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p \\
& V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \\
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \\
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
 & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \\
 \end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
 & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))))
 \end{aligned} \tag{67}$$

### Theorem 1

$$\begin{aligned}
 & (p (ap c_2Edivides_2Eprime (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
 & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))
 \end{aligned}$$