

# thm\_2Edivides\_2EprimePRIMES

(TMUZ9sNKRkhkaEdDrnDrWRiNcaqJJNUXCq)

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**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (1)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (2)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (3)$$

**Definition 4** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 5** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (4)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (5)$$

**Definition 6** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 7** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap c\_2Enum\_2EABS\_num (m))$

Let  $c_2$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

**Definition 8** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2EBIT1$

**Definition 9** We define c\_2Earithmetic\_2ENUMERAL to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x.$

**Definition 10** We define  $c \in \text{Emin} \rightarrow \text{3D\_3D\_3E}$  to be  $\lambda P \in 2.\lambda Q \in 2.\text{inj\_o } (p \ P \Rightarrow p \ Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c\_2Ebool\_2E\_21) 2)) (\lambda V2t \in$

Let  $c_2Earithmetic_2E_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

**Definition 12** We define  $c_2Ebool\_2E\_3F$  to be  $\lambda A.\lambda VOP \in (2^{A \rightarrow 27a}).(ap\ VOP\ (ap\ (c_2Emin\_2E\_40$

**Definition 13** We define  $c\_2Edivides\_2Edivides$  to be  $\lambda V0a \in ty\_2Enum\_2Enum. \lambda V1b \in ty\_2Enum\_2Enum.$

**Definition 14** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E_21\ 2)\ (\lambda V0t\in 2.V0t))$ .

**Definition 15** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E))$

**Definition 16** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t_1 \in 2.(\lambda V1t_2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 17** We define  $c\_2E\text{divides\_2Eprime}$  to be  $\lambda V0a \in t\forall .2Enum\_2Enum.(ap\ (ap\ c\_2E\text{bool\_2E\_2F\_5C}\$

**Definition 18** We define  $c$  2Earithmetic 2EBIT2 to be  $\lambda V0n \in tu\ 2Enum.\ 2Enum.(ap\ (ap\ c\ 2Earithmetic\ 2EBIT2)\ n)$

Let  $c$  2Edivides 2EPRIMES ; i be given. Assume the following.

$$c. 2E \text{ divides } 2EPBIMES \in (tu \cdot 2E \text{ enum } 2E \text{ enum }^{tu \cdot 2E \text{ enum } - 2E \text{ enum}}) \quad (8)$$

**Definition 19.** We define a 2Ecombin 2Eo to be  $\lambda A.27a : \iota \backslash \lambda A.27b$

$$c\_2Edivides\_2EPRIMES \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \quad (8)$$

**Definition 19** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1$

Let  $c_2Ewhile_2EWHILE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A \_27a. \text{nonempty } A \_27a \Rightarrow c \_2E \text{while\_2E} WHILE \ A \_27a \in (((A \_27a^{A \_27a})^{(A \_27a^{A \_27a})})^{(2^{A \_27a})}) \quad (9)$$

**Definition 20** We define  $c\_2Ewhile\_2ELEAST$  to be  $\lambda V0P \in (2^{ty\_2Enum\_2Enum}).(ap\ (ap\ (ap\ (ap\ (c\_2Ewhile\_2Ewhile\_2Eleast))))$

**Definition 21** We define  $c\_2Eprim\_rec\_2E\lambda C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. ((V0m = c\_2Enum\_2E0) \vee (\exists V1n \in ty\_2Enum\_2Enum. (V0m = (ap c\_2Enum\_2ESUC V1n))))) \quad (10)$$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (13)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t)) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (14)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (16)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee (p V1B))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge (\neg(p V1B)))))))) \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (18)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27))))))) \quad (19)$$

Assume the following.

$$(p (ap c_2Edivides_2Eprime (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))) \quad (20)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\exists V1p \in ty\_2Enum\_2Enum. (p (ap (ap c_2Eprim\_rec\_2E\_3C V0n) V1p)) \wedge (p (ap c_2Edivides_2Eprime V1p)))) \quad (21)$$

Assume the following.

$$(((ap c_2Edivides_2EPIMES c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \wedge (\forall V0n \in ty\_2Enum\_2Enum. ((ap c_2Edivides_2EPIMES (ap c_2Enum_2ESUC V0n)) = (ap c_2Ewhile_2ELEAST (\lambda V1p \in ty\_2Enum\_2Enum. (ap (ap c_2Ebool_2F_5C (ap c_2Edivides_2Eprime V1p)) (ap (ap c_2Eprim\_rec\_2E\_3C (ap c_2Edivides_2EPIMES V0n)) V1p))))))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (23)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (27)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \quad (28)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
 & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))) \\
 \end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
 & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \\
 \end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
 & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \\
 \end{aligned} \tag{31}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \tag{32}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \tag{33}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0Q \in (2^{ty\_2Enum\_2Enum}). (\forall V1P \in (2^{ty\_2Enum\_2Enum}). \\
 & (((\exists V2n \in ty\_2Enum\_2Enum. (p (ap V1P V2n))) \wedge (\forall V3n \in \\
 & ty\_2Enum\_2Enum. (((\forall V4m \in ty\_2Enum\_2Enum. ((p (ap (ap (ap c\_2Eprim\_rec\_2E\_3C \\
 & V4m) V3n)) \Rightarrow (\neg(p (ap V1P V4m)))) \wedge (p (ap V1P V3n)) \Rightarrow (p (ap V0Q V3n)))))) \Rightarrow \\
 & (p (ap V0Q (ap c\_2Ewhile\_2ELEAST V1P))))))) \\
 \end{aligned} \tag{34}$$

### Theorem 1

$$(\forall V0n \in ty\_2Enum\_2Enum. (p (ap c\_2Edivides\_2Eprime (ap c\_2Edivides\_2EPRIMES \\
 V0n)))) \tag{35}$$