thm_2Eenumeral_2EIN__bt__to__set (TMT2QpmPR945MxLsDdCNtu1bN7VuC9moDAi)

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Let $ty\_2EternaryComparisons\_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty\_2EternaryComparisons\_2Eordering \tag{1}$$

Let $c\_2EternaryComparisons\_2ELESS : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2ELESS \in ty\_2EternaryComparisons\_2Eordering \tag{2}$$

Let $ty\_2Etoto\_2Etoto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etoto\_2Etoto\ A0) \tag{3}$$

Let $c\_2Etoto\_2Eapto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etoto\_2Eapto\ A\_27a \in (((ty\_2EternaryComparisons\_2Eordering^{A\_27a})^{A\_2} \tag{4}$$

Let $ty\_2Eenumeral\_2Ebt : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eenumeral\_2Ebt\ A0) \tag{5}$$

Let $c\_2Eenumeral\_2Enode : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eenumeral\_2Enode\ A\_27a \in ((((ty\_2Eenumeral\_2Ebt\ A\_27a)^{(ty\_2Eenumeral\_2Ebt\ A\_27a)})^{A\_27a})^{(ty\_2Eenumeral\_2Ebt\ A\_27a)}) \tag{6}$$

Let $c\_2Eenumeral\_2Ent : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eenumeral\_2Ent\ A\_27a \in (ty\_2Eenumeral\_2Ebt\ A\_27a) \tag{7}$$

Let $c\_2Eenumeral\_2EENUMERAL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eenumeral\_2EENUMERAL\ A\_27a \in (((2^{A\_27a})^{(ty\_2Eenumeral\_2Ebt\ A\_27a)})^{(ty\_2Etoto\_2Etoto\ A\_27a)}) \tag{8}$$

Definition 1 *We define* $\mathsf{c\_2Emin\_2E\_3D}$ *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* $\mathsf{c\_2Ebool\_2ET}$ *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{2}))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* $\mathsf{c\_2Emin\_2E\_3D\_3D\_3E}$ *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 4 *We define* $\mathsf{c\_2Ebool\_2E\_21}$ *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* $\mathsf{c\_2Ebool\_2E\_2F\_5C}$ *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{9}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{10}$$

Definition 6 *We define* $\mathsf{c\_2Epair\_2E\_2C}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Definition 7 *We define* $\mathsf{c\_2Emin\_2E\_40}$ *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 8 *We define* $\mathsf{c\_2Ebool\_2E\_3F}$ *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{11}$$

Definition 9 *We define* $\mathsf{c\_2Ebool\_2EF}$ *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 10 *We define* $\mathsf{c\_2Epred\_\_set\_2EEMPTY}$ *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2EF})$.

Definition 11 *We define* $\mathsf{c\_2Ebool\_2EIN}$ *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 12 *We define* $\mathsf{c\_2Ebool\_2E\_5C\_2F}$ *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 13 *We define* $\mathsf{c\_2Epred\_\_set\_2EUNION}$ *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c$

Definition 14 *We define* $\mathsf{c\_2Epred\_\_set\_2EINSERT}$ *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c$

Definition 15 *We define* $\mathsf{c\_2Ebool\_2E\_7E}$ *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Assume the following.

$$True \tag{12}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{13}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{14}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{15}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t))))))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0cmp \in (ty\_2Etoto\_2Etoto\ A\_27a).((ap\ (ap\ (c\_2Eenumeral\_2EENUMERAL\ A\_27a)\ V0cmp)\ (c\_2Eenumeral\_2Ent\ A\_27a)) = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))) \wedge (\forall V1cmp \in (ty\_2Etoto\_2Etoto\ A\_27a).(\forall V2l \in (ty\_2Eenumeral\_2Ebt\ A\_27a).(\forall V3x \in A\_27a.(\forall V4r \in (ty\_2Eenumeral\_2Ebt\ A\_27a).((ap\ (ap\ (c\_2Eenumeral\_2EENUMERAL\ A\_27a)\ V1cmp)\ (ap\ (ap\ (ap\ (c\_2Eenumeral\_2Enode\ A\_27a)\ V2l)\ V3x)\ V4r)) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27a)\ (\lambda V5y \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V5y)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V5y)\ (ap\ (ap\ (c\_2Eenumeral\_2EENUMERAL\ A\_27a)\ V1cmp)\ V2l)))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ ty\_2EternaryComparisons\_2Eordering)\ (ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V1cmp)\ V5y)\ V3x))\ c\_2EternaryComparisons\_2ELESS))))))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V3x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27a)\ (\lambda V6z \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V6z)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V6z)\ (ap\ (ap\ (c\_2Eenumeral\_2EENUMERAL\ A\_27a)\ V1cmp)\ V4r)))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ ty\_2EternaryComparisons\_2Eordering)\ (ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V1cmp)\ V3x)\ V6z))\ c\_2EternaryComparisons\_2ELESS)))))))))))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2a \in A\_27a.(\forall V3b \in A\_27b.(((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}).(\forall V1v \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1v)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V1v)\ \mathsf{c\_2Ebool\_2ET}) = (ap\ V0f\ V2x)))))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\neg(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \vee (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(\forall V2s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V1y)\ V2s))) \Leftrightarrow ((V0x = V1y) \lor (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ V2s))))))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{25}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{26}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \lor (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{27}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \lor (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{28}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{29}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \lor ((p\ V1q) \lor (p\ V2r))) \land (((p\ V0p) \lor ((\neg(p\ V2r)) \lor (\neg(p\ V1q)))) \land (((p\ V1q) \lor ((\neg(p\ V2r)) \lor (\neg(p\ V0p)))) \land ((p\ V2r) \lor ((\neg(p\ V1q)) \lor (\neg(p\ V0p))))))))))) \tag{30}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \lor (p\ V2r))) \Leftrightarrow (((p\ V0p) \lor (\neg(p\ V1q))) \land (((p\ V0p) \lor (\neg(p\ V2r))) \land ((p\ V1q) \lor ((p\ V2r) \lor (\neg(p\ V0p)))))))))) \tag{31}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p) \Leftrightarrow (\neg(p\ V1q))) \Leftrightarrow (((p\ V0p) \lor (p\ V1q)) \land ((\neg(p\ V1q)) \lor (\neg(p\ V0p))))))) \tag{32}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0cmp \in (ty\_2Etoto\_2Etoto\ A\_27a).(\forall V1y \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1y)\ (ap\ (ap\ (c\_2Eenumeral\_2EENUMERAL\ A\_27a)\ V0cmp)\ (c\_2Eenumeral\_2Ent\ A\_27a)))) \Leftrightarrow False))) \wedge (\forall V2cmp \in (ty\_2Etoto\_2Etoto\ A\_27a).(\forall V3l \in (ty\_2Eenumeral\_2Ebt\ A\_27a).(\forall V4x \in A\_27a.(\forall V5r \in (ty\_2Eenumeral\_2Ebt\ A\_27a).(\forall V6y \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V6y)\ (ap\ (ap\ (c\_2Eenumeral\_2EENUMERAL\ A\_27a)\ V2cmp)\ (ap\ (ap\ (ap\ (c\_2Eenumeral\_2Enode\ A\_27a)\ V3l)\ V4x)\ V5r)))) \Leftrightarrow (((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V6y)\ (ap\ (ap\ (c\_2Eenumeral\_2EENUMERAL\ A\_27a)\ V2cmp)\ V3l))) \wedge ((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V2cmp)\ V6y)\ V4x) = c\_2EternaryComparisons\_2ELESS)) \vee ((V6y = V4x) \vee ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V6y)\ (ap\ (ap\ (c\_2Eenumeral\_2EENUMERAL\ A\_27a)\ V2cmp)\ V5r))) \wedge ((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V2cmp)\ V4x)\ V6y) = c\_2EternaryComparisons\_2ELESS)))))))))))$$