thm_2Eenumeral_2EOWL__bt__to__ol (TMJbLag6CcDNpg8sJL5H5hnjNybAGcChbWu)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27a}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $ty\_2Eenumeral\_2Ebt : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eenumeral\_2Ebt\ A0) \tag{1}$$

Let $ty\_2Etoto\_2Etoto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etoto\_2Etoto\ A0) \tag{2}$$

Let $c\_2Eenumeral\_2EENUMERAL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eenumeral\_2EENUMERAL\ A\_27a \in (((2^{A\_27a})^{(ty\_2Eenumeral\_2Ebt\ A\_27a)})^{(ty\_2Etoto\_2Etoto\ A\_27a)}) \tag{3}$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{4}$$

Let $c\_2Eenumeral\_2Ebt\_\_to\_\_ol : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eenumeral\_2Ebt\_\_to\_\_ol\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Eenumeral\_2Ebt\ A\_27a)})^{(ty\_2Etoto\_2Etoto\ A\_27a)}) \tag{5}$$

Let $c\_2Eenumeral\_2EOL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eenumeral\_2EOL\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Etoto\_2Etoto\ A\_27a)} \tag{6}$$

Let $c\_2Elist\_2ELIST\_\_TO\_\_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a \in ((2^{A\_27a})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{7}$$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 8 *We define* c_2Eenumeral_2EOWL *to be* $\lambda A\_27a : \iota.\lambda V0cmp \in (ty\_2Etoto\_2Etoto\ A\_27a).\lambda V1s$

Assume the following.

$$True \tag{8}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{10}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{11}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0cmp \in (ty\_2Etoto\_2Etoto\ A\_27a).(\forall V1t \in (ty\_2Eenumeral\_2Ebt\ A\_27a).((ap\ (ap\ (c\_2Eenumeral\_2EENUMERAL\ A\_27a)\ V0cmp)\ V1t) = (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ (ap\ (ap\ (c\_2Eenumeral\_2Ebt\_\_to\_\_ol\ A\_27a)\ V0cmp)\ V1t))))) \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0cmp \in (ty\_2Etoto\_2Etoto\ A\_27a).(\forall V1t \in (ty\_2Eenumeral\_2Ebt\ A\_27a).(p\ (ap\ (ap\ (c\_2Eenumeral\_2EOL\ A\_27a)\ V0cmp)\ (ap\ (ap\ (c\_2Eenumeral\_2Ebt\_\_to\_\_ol\ A\_27a)\ V0cmp)\ V1t))))) \tag{13}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0cmp \in (ty\_2Etoto\_2Etoto\ A\_27a).(\forall V1t \in (ty\_2Eenumeral\_2Ebt\ A\_27a).(p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eenumeral\_2EOWL}\ A\_27a)\ V0cmp)\ (ap\ (ap\ (c\_2Eenumeral\_2EENUMERAL\ A\_27a)\ V0cmp)\ V1t))\ (ap\ (ap\ (c\_2Eenumeral\_2Ebt\_\_to\_\_ol\ A\_27a)\ V0cmp)\ V1t)))))$$