thm_2Eenumeral_2Eincr__smerge__ind (TMXFVXYBPVk-davP3PD5p5FrQFWTFS8MyLUL)

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Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

Let $ty\_2Etoto\_2Etoto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etoto\_2Etoto\ A0) \tag{2}$$

Let $c\_2Eenumeral\_2Esmerge : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eenumeral\_2Esmerge\ A\_27a \in ($$
$$(((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Etoto\_2Etoto\ A\_27a)}) \tag{3}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{4}$$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{5}$$

Let $c\_2EbasicSize\_2Eoption\_\_size : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2EbasicSize\_2Eoption\_\_size$$
$$A\_27a \in ((ty\_2Enum\_2Enum^{(ty\_2Eoption\_2Eoption\ A\_27a)})^{(ty\_2Enum\_2Enum^{A\_27a})}) \tag{6}$$

Let $c\_2Elist\_2Elist\_\_size : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2Elist\_\_size\ A\_27a \in ($$
$$(ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Enum\_2Enum^{A\_27a})}) \tag{7}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist$$
$$A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{8}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist$$
$$A\_27a)$$

$$(9)$$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \qquad (10)$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \qquad (11)$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \text{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \text{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \qquad (12)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \qquad (13)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (14)$$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ \text{c\_2Enum\_2EABS\_\_num}$ (

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\textbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \textbf{then}\ (the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\text{c\_2Emin\_2E\_40}$

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 12 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

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Definition 13 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 14 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (15)$$

Definition 15 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP).$

Definition 16 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 17 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap$ (c_2Ebool_2E

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(16)$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(17)$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(18)$$

Definition 18 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Definition 19 *We define* c_2Enumeral_2EiiSUC *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap$ c_2Enum_2ESUC $(ap$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(19)$$

Definition 20 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Definition 21 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 22 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 23 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Definition 24 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty \ ty\_2Eone\_2Eone \tag{20}$$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty \ A0 \Rightarrow \forall A1.nonempty \ A1 \Rightarrow nonempty \ (ty\_2Esum\_2Esum \\ A0 \ A1) \tag{21}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum \\ A\_27a \ A\_27b \in ((ty\_2Esum\_2Esum \ A\_27a \ A\_27b)^{(((2^{A\_27b})^{A\_27a})^2)}) \tag{22}$$

Definition 25 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap \ (c\_2Esum\_2EABS$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS \ A\_27a \in \\ ((ty\_2Eoption\_2Eoption \ A\_27a)^{(ty\_2Esum\_2Esum \ A\_27a \ ty\_2Eone\_2Eone)}) \tag{23}$$

Definition 26 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap \ (c\_2Eoption\_2Eoption\_$

Definition 27 *We define* c_2Eone_2Eone *to be* $(ap \ (c\_2Emin\_2E\_40 \ ty\_2Eone\_2Eone) \ (\lambda V0x \in ty\_2Eone\_2$

Definition 28 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap \ (c\_2Esum\_2EABS$

Definition 29 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap \ (c\_2Eoption\_2Eoption\_\_ABS \ A\_27a) \ (a$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty \ A0 \Rightarrow \forall A1.nonempty \ A1 \Rightarrow nonempty \ (ty\_2Epair\_2Eprod \\ A0 \ A1) \tag{24}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a \ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod \ A\_27a \ A\_27b)}) \tag{25}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow c\_2Epair\_2ESND \\ A\_27a \ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod \ A\_27a \ A\_27b)}) \tag{26}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod \\ A\_27a \ A\_27b \in ((ty\_2Epair\_2Eprod \ A\_27a \ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{27}$$

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Definition 30 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Definition 31 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Definition 32 *We define* c_2Erelation_2Einv__image *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27b})^{A\_27b}).\lambda V$

Definition 33 *We define* c_2Erelation_2EWF *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap\ (\text{c\_2Ebool\_2E\_21}$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \text{c\_2Enum\_2E0})\ V0m) = V0m) \wedge (((ap\ (\\ ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ \text{c\_2Enum\_2E0}) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2B\\ (ap\ \text{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ \text{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\\ V0m)\ V1n))) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ \text{c\_2Enum\_2ESUC}\\ V1n)) = (ap\ \text{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))))))))$$

$$(28)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\\ V1n)\ V0m))))$$

$$(29)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\\ V1n)\ V0m))))$$

$$(30)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ \forall V2p \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2p)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ V2p)))))$$

$$(31)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3C\_3D}\\ \text{c\_2Enum\_2E0})\ V0n)))$$

$$(32)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ (\neg(p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3C\_3D}\\ V1n)\ V0m)))))$$

$$(33)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\
((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \textsf{c\_2Enum\_2E0})\ V0m) = \textsf{c\_2Enum\_2E0}) \land \\
(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ \textsf{c\_2Enum\_2E0}) = \textsf{c\_2Enum\_2E0}) \land \\
(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL} \\
(ap\ \textsf{c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO})))\ V0m) = V0m) \land \\
(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL} \\
(ap\ \textsf{c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO}))) = V0m) \land (\\
((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \textsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap \\
(ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n)) \\
V1n)) \land ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \textsf{c\_2Enum\_2ESUC}\ V1n)) = \\
(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A \\
V0m)\ V1n)))))))))))) \tag{34}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\
\forall V2p \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D} \\
V0m)\ V1n)) \land (p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p))) \Rightarrow (p\ (\\
ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V2p)))))) \tag{35}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\
\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D} \\
(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
V0m)\ V2p))) \Leftrightarrow (p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p)))))) \tag{36}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ \textsf{c\_2Enum\_2ESUC}\ V0n) = (ap\ (ap \\
c\_2Earithmetic\_2E\_2B\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1} \\
\textsf{c\_2Earithmetic\_2EZERO}))\ V0n))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0f \in (ty\_2Enum\_2Enum^{A\_27a}). \\
((ap\ (ap\ (c\_2EbasicSize\_2Eoption\_\_size\ A\_27a)\ V0f)\ (\textsf{c\_2Eoption\_2ENONE} \\
A\_27a)) = \textsf{c\_2Enum\_2E0}) \land (\forall V1f \in (ty\_2Enum\_2Enum^{A\_27a}). \\
(\forall V2x \in A\_27a.((ap\ (ap\ (c\_2EbasicSize\_2Eoption\_\_size\ A\_27a) \\
V1f)\ (ap\ (\textsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2x)) = (ap\ \textsf{c\_2Enum\_2ESUC}\ (ap \\
V1f\ V2x)))))) \tag{38}$$

Assume the following.

$$True \tag{39}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{40}$$

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Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{41}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t))))))))) \tag{42}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True))) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\ A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{44}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ p\ V0t)))))))) \tag{45}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0f \in (ty\_2Enum\_2Enum^{A\_27a}). \\ ((ap\ (ap\ (c\_2Elist\_2Elist\_\_size\ A\_27a)\ V0f)\ (c\_2Elist\_2ENIL\ A\_27a)) = \\ \mathsf{c\_2Enum\_2E0})) \wedge (\forall V1f \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V2a0 \in \\ A\_27a.(\forall V3a1 \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2Elist\_\_size \\ A\_27a)\ V1f)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2a0)\ V3a1)) = (ap\ (ap \\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\ \mathsf{c\_2Earithmetic\_2EZERO})))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ V1f \\ V2a0))\ (ap\ (ap\ (c\_2Elist\_2Elist\_\_size\ A\_27a)\ V1f)\ V3a1))))))))) \tag{47}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist \\ A\_27a).((V0l = (c\_2Elist\_2ENIL\ A\_27a)) \vee (\exists V1h \in A\_27a.(\\ \exists V2t \in (ty\_2Elist\_2Elist\ A\_27a).(V0l = (ap\ (ap\ (c\_2Elist\_2ECONS \\ A\_27a)\ V1h)\ V2t)))))) \tag{48}$$

Assume the following.

$$((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$$

c_2Enum_2E0) $V0n) = V0n))\wedge((\forall V1n \in ty\_2Enum\_2Enum.((ap$

$(ap\ c\_2Earithmetic\_2E\_2B\ V1n)$ c_2Enum_2E0) $= V1n))\wedge((\forall V2n \in$

$ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$

$(ap$ c_2Earithmetic_2ENUMERAL $V2n))$ $(ap$ c_2Earithmetic_2ENUMERAL

$V3m)) = (ap$ c_2Earithmetic_2ENUMERAL $(ap$ c_2Enumeral_2EiZ $(ap$

$(ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ V3m))))))\wedge((\forall V4n \in ty\_2Enum\_2Enum.$

$((ap\ (ap\ c\_2Earithmetic\_2E\_2A$ c_2Enum_2E0) $V4n) =$ c_2Enum_2E0))$\wedge$

$((\forall V5n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A$

$V5n)$ c_2Enum_2E0) $=$ c_2Enum_2E0))$\wedge((\forall V6n \in ty\_2Enum\_2Enum.$

$(\forall V7m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ ($

ap c_2Earithmetic_2ENUMERAL $V6n))$ $(ap$ c_2Earithmetic_2ENUMERAL

$V7m)) = (ap$ c_2Earithmetic_2ENUMERAL $(ap\ (ap\ c\_2Earithmetic\_2E\_2A$

$V6n)\ V7m)))))\wedge((\forall V8n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$

c_2Enum_2E0) $V8n) =$ c_2Enum_2E0))$\wedge((\forall V9n \in ty\_2Enum\_2Enum.$

$((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V9n)$ c_2Enum_2E0) $= V9n))\wedge((\forall V10n \in$

$ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$

$(ap$ c_2Earithmetic_2ENUMERAL $V10n))$ $(ap$ c_2Earithmetic_2ENUMERAL

$V11m)) = (ap$ c_2Earithmetic_2ENUMERAL $(ap\ (ap\ c\_2Earithmetic\_2E\_2D$

$V10n)\ V11m)))))\wedge((\forall V12n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP$

c_2Enum_2E0) $(ap$ c_2Earithmetic_2ENUMERAL $(ap$ c_2Earithmetic_2EBIT1

$V12n))) =$ c_2Enum_2E0))$\wedge((\forall V13n \in ty\_2Enum\_2Enum.((ap$

$(ap\ c\_2Earithmetic\_2EEXP$ c_2Enum_2E0) $(ap$ c_2Earithmetic_2ENUMERAL

$(ap$ c_2Earithmetic_2EBIT2 $V13n))) =$ c_2Enum_2E0))$\wedge((\forall V14n \in$

$ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V14n)$ c_2Enum_2E0) $=$

$(ap$ c_2Earithmetic_2ENUMERAL $(ap$ c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))$\wedge$

$((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.$

$((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap$ c_2Earithmetic_2ENUMERAL $V15n))$

$(ap$ c_2Earithmetic_2ENUMERAL $V16m)) = (ap$ c_2Earithmetic_2ENUMERAL

$(ap\ (ap\ c\_2Earithmetic\_2EEXP\ V15n)\ V16m)))))\wedge(((ap$ c_2Enum_2ESUC

c_2Enum_2E0) $= (ap$ c_2Earithmetic_2ENUMERAL $(ap$ c_2Earithmetic_2EBIT1

c_2Earithmetic_2EZERO)))$\wedge((\forall V17n \in ty\_2Enum\_2Enum.($

$(ap$ c_2Enum_2ESUC $(ap$ c_2Earithmetic_2ENUMERAL $V17n)) = (ap$ c_2Earithmetic_2ENUMERAL

$(ap$ c_2Enum_2ESUC $V17n))))\wedge(((ap$ c_2Eprim__rec_2EPRE c_2Enum_2E0) $=$

c_2Enum_2E0)$\wedge((\forall V18n \in ty\_2Enum\_2Enum.((ap$ c_2Eprim__rec_2EPRE

$(ap$ c_2Earithmetic_2ENUMERAL $V18n)) = (ap$ c_2Earithmetic_2ENUMERAL

$(ap$ c_2Eprim__rec_2EPRE $V18n))))\wedge((\forall V19n \in ty\_2Enum\_2Enum.$

$(((ap$ c_2Earithmetic_2ENUMERAL $V19n) =$ c_2Enum_2E0)$\Leftrightarrow(V19n =$ c_2Earithmetic_2EZERO)))$\wedge$

$((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap$ c_2Earithmetic_2ENUMERAL

$V20n))\Leftrightarrow(V20n =$ c_2Earithmetic_2EZERO)))$\wedge((\forall V21n \in ty\_2Enum\_2Enum.$

$(\forall V22m \in ty\_2Enum\_2Enum.(((ap$ c_2Earithmetic_2ENUMERAL

$V21n) = (ap$ c_2Earithmetic_2ENUMERAL $V22m))\Leftrightarrow(V21n = V22m))))\wedge$

$((\forall V23n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap$ c_2Eprim__rec_2E_3C

$V23n)$ c_2Enum_2E0))$\Leftrightarrow False))\wedge((\forall V24n \in ty\_2Enum\_2Enum.$

$((p\ (ap\ (ap$ c_2Eprim__rec_2E_3C c_2Enum_2E0) $(ap$ c_2Earithmetic_2ENUMERAL

$V24n)))\Leftrightarrow(p\ (ap\ (ap$ c_2Eprim__rec_2E_3C c_2Earithmetic_2EZERO)

$V24n))))\wedge((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum.$

$((p\ (ap\ (ap$ c_2Eprim__rec_2E_3C $(ap$ c_2Earithmetic_2ENUMERAL

$V25n))$ $(ap$ c_2Earithmetic_2ENUMERAL $V26m)))\Leftrightarrow(p\ (ap\ (ap$ c_2Eprim__rec_2E_3C

$V25n)\ V26m)))))\wedge((\forall V27n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap$ c_2Earithmetic_2E_3E

c_2Enum_2E0) $V27n))\Leftrightarrow False))\wedge((\forall V28n \in ty\_2Enum\_2Enum.$

$((p\ (ap\ (ap$ c_2Earithmetic_2E_3E $(ap$ c_2Earithmetic_2ENUMERAL

$V28n))$ c_2Enum_2E0))$\Leftrightarrow(p\ (ap\ (ap$ c_2Eprim__rec_2E_3C c_2Earithmetic_2EZERO)

$V28n))))\wedge((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum.$

$((p\ (ap\ (ap$ c_2Earithmetic_2E_3E $(ap$ c_2Earithmetic_2ENUMERAL

$V29n))$ $(ap$ c_2Earithmetic_2ENUMERAL $V30m)))\Leftrightarrow(p\ (ap\ (ap$ c_2Eprim__rec_2E_3C

$V30m)\ V29n)))))\wedge((\forall V31n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap$ c_2Earithmetic_2E_3C_3D

c_2Enum_2E0) $V31n))\Leftrightarrow True))\wedge((\forall V32n \in ty\_2Enum\_2Enum.$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$((ap \ \mathsf{c\_2Enumeral\_2EiZ} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B \ \mathsf{c\_2Earithmetic\_2EZERO})$$
$$V0n)) = V0n) \wedge (((ap \ \mathsf{c\_2Enumeral\_2EiZ} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B$$
$$V0n) \ \mathsf{c\_2Earithmetic\_2EZERO})) = V0n) \wedge (((ap \ \mathsf{c\_2Enumeral\_2EiZ} \ ($$
$$ap \ (ap \ c\_2Earithmetic\_2E\_2B \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1} \ V0n)) \ ($$
$$ap \ \mathsf{c\_2Earithmetic\_2EBIT1} \ V1m))) = (ap \ \mathsf{c\_2Earithmetic\_2EBIT2} \ ($$
$$ap \ \mathsf{c\_2Enumeral\_2EiZ} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B \ V0n) \ V1m)))) \wedge$$
$$(((ap \ \mathsf{c\_2Enumeral\_2EiZ} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n)) \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT2} \ V1m))) = (ap \ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$(ap \ \mathsf{c\_2Enum\_2ESUC} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B \ V0n) \ V1m)))) \wedge ($$
$$((ap \ \mathsf{c\_2Enumeral\_2EiZ} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n)) \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1} \ V1m))) = (ap \ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$(ap \ \mathsf{c\_2Enum\_2ESUC} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B \ V0n) \ V1m)))) \wedge ($$
$$((ap \ \mathsf{c\_2Enumeral\_2EiZ} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n)) \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT2} \ V1m))) = (ap \ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$(ap \ \mathsf{c\_2Enum\_2ESUC} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B \ V0n) \ V1m)))) \wedge ($$
$$((ap \ \mathsf{c\_2Enum\_2ESUC} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B \ \mathsf{c\_2Earithmetic\_2EZERO})$$
$$V0n)) = (ap \ \mathsf{c\_2Enum\_2ESUC} \ V0n)) \wedge (((ap \ \mathsf{c\_2Enum\_2ESUC} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B$$
$$V0n) \ \mathsf{c\_2Earithmetic\_2EZERO})) = (ap \ \mathsf{c\_2Enum\_2ESUC} \ V0n)) \wedge (((ap$$
$$\mathsf{c\_2Enum\_2ESUC} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n)) \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1} \ V1m))) = (ap \ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$(ap \ \mathsf{c\_2Enum\_2ESUC} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B \ V0n) \ V1m)))) \wedge ($$
$$((ap \ \mathsf{c\_2Enum\_2ESUC} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n)) \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT2} \ V1m))) = (ap \ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$(ap \ \mathsf{c\_2Enum\_2ESUC} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B \ V0n) \ V1m)))) \wedge ($$
$$((ap \ \mathsf{c\_2Enum\_2ESUC} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n)) \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1} \ V1m))) = (ap \ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$(ap \ \mathsf{c\_2Enum\_2ESUC} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B \ V0n) \ V1m)))) \wedge ($$
$$((ap \ \mathsf{c\_2Enum\_2ESUC} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n)) \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT2} \ V1m))) = (ap \ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$(ap \ \mathsf{c\_2Enumeral\_2EiiSUC} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B \ V0n) \ V1m)))) \wedge$$
$$(((ap \ \mathsf{c\_2Enumeral\_2EiiSUC} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B \ \mathsf{c\_2Earithmetic\_2EZERO})$$
$$V0n)) = (ap \ \mathsf{c\_2Enumeral\_2EiiSUC} \ V0n)) \wedge (((ap \ \mathsf{c\_2Enumeral\_2EiiSUC}$$
$$(ap \ (ap \ c\_2Earithmetic\_2E\_2B \ V0n) \ \mathsf{c\_2Earithmetic\_2EZERO})) = ($$
$$ap \ \mathsf{c\_2Enumeral\_2EiiSUC} \ V0n)) \wedge (((ap \ \mathsf{c\_2Enumeral\_2EiiSUC} \ (ap \ ($$
$$ap \ c\_2Earithmetic\_2E\_2B \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1} \ V0n)) \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V1m))) = (ap \ \mathsf{c\_2Earithmetic\_2EBIT2} \ (ap \ \mathsf{c\_2Enum\_2ESUC} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B$$
$$V0n) \ V1m)))) \wedge (((ap \ \mathsf{c\_2Enumeral\_2EiiSUC} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B$$
$$(ap \ \mathsf{c\_2Earithmetic\_2EBIT1} \ V0n)) \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT2} \ V1m))) =$$
$$(ap \ \mathsf{c\_2Earithmetic\_2EBIT1} \ (ap \ \mathsf{c\_2Enumeral\_2EiiSUC} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B$$
$$V0n) \ V1m)))) \wedge (((ap \ \mathsf{c\_2Enumeral\_2EiiSUC} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B$$
$$(ap \ \mathsf{c\_2Earithmetic\_2EBIT2} \ V0n)) \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1} \ V1m))) =$$
$$(ap \ \mathsf{c\_2Earithmetic\_2EBIT1} \ (ap \ \mathsf{c\_2Enumeral\_2EiiSUC} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B$$
$$V0n) \ V1m)))) \wedge ((ap \ \mathsf{c\_2Enumeral\_2EiiSUC} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B$$
$$(ap \ \mathsf{c\_2Earithmetic\_2EBIT2} \ V0n)) \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT2} \ V1m))) =$$
$$(ap \ \mathsf{c\_2Earithmetic\_2EBIT2} \ (ap \ \mathsf{c\_2Enumeral\_2EiiSUC} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B$$
$$V0n) \ V1m)))))))))))))))))))))))$$

$$(50)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(\\
((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Earithmetic\_2EZERO})\ V0n)) \Leftrightarrow \\
True) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
V0n))\ \mathsf{c\_2Earithmetic\_2EZERO})) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ \mathsf{c\_2Earithmetic\_2EZERO})) \Leftrightarrow False) \wedge \\
(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \\
V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \\
V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) \Leftrightarrow (\neg (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \\
V1m)\ V0n)))) \wedge ((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \\
V0n)\ V1m))))))))))))$$

(51)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0opt \in (ty\_2Eoption\_2Eoption \\
A\_27a).((V0opt = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \vee (\exists V1x \in A\_27a. \\
(V0opt = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x)))))$$

(52)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\\
\forall V0x \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(\exists V1q \in A\_27a. \\
(\exists V2r \in A\_27b.(V0x = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b) \\
V1q)\ V2r)))))$$

(53)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\\
\forall V0x \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C} \\
A\_27a\ A\_27b)\ (ap\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b)\ V0x))\ (ap\ (c\_2Epair\_2ESND \\
A\_27a\ A\_27b)\ V0x)) = V0x))$$

(54)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\\
\forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (c\_2Epair\_2ESND\ A\_27a \\
A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y)) = V1y)))$$

(55)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}).(\forall V1x \in \\
A\_27a.(\forall V2y \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a \\
A\_27b\ A\_27c)\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V1x)\ V2y)) = \\
(ap\ (ap\ V0f\ V1x)\ V2y)))))$$

(56)

Assume the following.

$$(p \ (ap \ (\mathsf{c\_2Erelation\_2EWF} \ ty\_2Enum\_2Enum) \ \mathsf{c\_2Eprim\_rec\_2E\_3C})) \quad (57)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).$$
$$((p \ (ap \ (\mathsf{c\_2Erelation\_2EWF} \ A\_27a) \ V0R)) \Rightarrow (\forall V1P \in (2^{A\_27a}).$$
$$((\forall V2x \in A\_27a.((\forall V3y \in A\_27a.((p \ (ap \ (ap \ V0R \ V3y) \ V2x)) \Rightarrow \quad (58)$$
$$(p \ (ap \ V1P \ V3y)))) \Rightarrow (p \ (ap \ V1P \ V2x)))) \Rightarrow (\forall V4x \in A\_27a.(p \ (ap \ V1P \ V4x)))))))$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow ($$
$$\forall V0R \in ((2^{A\_27b})^{A\_27b}).(\forall V1f \in (A\_27b^{A\_27a}).((\$$
$$p \ (ap \ (\mathsf{c\_2Erelation\_2EWF} \ A\_27b) \ V0R)) \Rightarrow (p \ (ap \ (\mathsf{c\_2Erelation\_2EWF} \$$
$$A\_27a) \ (ap \ (ap \ (\mathsf{c\_2Erelation\_2Einv\_image} \ A\_27a \ A\_27b) \ V0R) \ V1f))))))$$
$$(59)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \quad (60)$$

Assume the following.

$$(\forall V0A \in 2.((p \ V0A) \Rightarrow ((\neg(p \ V0A)) \Rightarrow False))) \quad (61)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \lor (p \ V1B))) \Rightarrow False) \Leftrightarrow$$
$$((((p \ V0A) \Rightarrow False) \Rightarrow ((\neg(p \ V1B)) \Rightarrow False))))) \quad (62)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p \ V0A)) \lor (p \ V1B))) \Rightarrow False) \Leftrightarrow$$
$$(((p \ V0A) \Rightarrow ((\neg(p \ V1B)) \Rightarrow False))))) \quad (63)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p \ V0A)) \Rightarrow False) \Rightarrow (((p \ V0A) \Rightarrow False) \Rightarrow False))) \quad (64)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow ($$
$$(p \ V1q) \Leftrightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \lor ((p \ V1q) \lor (p \ V2r))) \land (((p \ V0p) \lor ((\neg($$
$$p \ V2r)) \lor (\neg(p \ V1q)))) \land (((p \ V1q) \lor ((\neg(p \ V2r)) \lor (\neg(p \ V0p)))) \land ((p \ V2r) \lor$$
$$((\neg(p \ V1q)) \lor (\neg(p \ V0p)))))))))))))$$
$$(65)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \lor$$
$$(p \ V1q)) \land ((\neg(p \ V1q)) \lor (\neg(p \ V0p)))))))$$
$$(66)$$

Theorem 1

$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (((2^{(ty\_2Elist\_2Elist\ (ty\_2Eoption\_2Eoption\ (ty\_2Elist\_2Elist\ A\_27a))))})^{(ty\_2E}$

$(((\forall V1cmp \in (ty\_2Etoto\_2Etoto\ A\_27a).(\forall V2l \in (ty\_2Elist\_2Elist$

$A\_27a).(p\ (ap\ (ap\ (ap\ V0P\ V1cmp)\ V2l)\ (c\_2Elist\_2ENIL\ (ty\_2Eoption\_$

$(ty\_2Elist\_2Elist\ A\_27a)))))))) \wedge ((\forall V3cmp \in (ty\_2Etoto\_2Etoto$

$A\_27a).(\forall V4l \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V5lol \in$

$(ty\_2Elist\_2Elist\ (ty\_2Eoption\_2Eoption\ (ty\_2Elist\_2Elist$

$A\_27a))).(p\ (ap\ (ap\ (ap\ V0P\ V3cmp)\ V4l)\ (ap\ (ap\ (c\_2Elist\_2ECO\_$

$(ty\_2Eoption\_2Eoption\ (ty\_2Elist\_2Elist\ A\_27a)))\ (c\_2Eoption\_2ENO$

$(ty\_2Elist\_2Elist\ A\_27a)))\ V5lol))))))) \wedge (\forall V6cmp \in (ty\_2Etoto\_2Eto$

$A\_27a).(\forall V7l \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V8m \in$

$(ty\_2Elist\_2Elist\ A\_27a).(\forall V9lol \in (ty\_2Elist\_2Elist$

$(ty\_2Eoption\_2Eoption\ (ty\_2Elist\_2Elist\ A\_27a))).((p\ (ap\ (ap\ (ap$

$(ap\ V0P\ V6cmp)\ (ap\ (ap\ (ap\ (c\_2Eenumeral\_2Esmerge\ A\_27a)\ V6c$

$V7l)\ V8m))\ V9lol)) \Rightarrow (p\ (ap\ (ap\ (ap\ V0P\ V6cmp)\ V7l)\ (ap\ (ap\ (c\_2Elist\_$

$(ty\_2Eoption\_2Eoption\ (ty\_2Elist\_2Elist\ A\_27a)))\ (ap\ (c\_2Eoption\_2ES$

$(ty\_2Elist\_2Elist\ A\_27a))\ V8m))\ V9lol))))))))))) \Rightarrow (\forall V10v \in$

$(ty\_2Etoto\_2Etoto\ A\_27a).(\forall V11v1 \in (ty\_2Elist\_2Elist$

$A\_27a).(\forall V12v2 \in (ty\_2Elist\_2Elist\ (ty\_2Eoption\_2Eoption$

$(ty\_2Elist\_2Elist\ A\_27a))).(p\ (ap\ (ap\ (ap\ V0P\ V10v)\ V11v1)\ V12v2)$