

thm_2ErrorStateMonad_2EES__CHOICE__FAIL__LID
 (TMKKfJ-
 taTHRv5wTx1xnP2FVBSX4GVJEyMBh)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$.
 Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 4 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone.V0x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (ap (c_2Emin_2E_40 P)) (ap (c_2Emin_2E_40 P)))))$

Definition 6 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \tag{3}$$

Definition 10 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap (c_2Esum_2EABS$
Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (4)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (5)$$

Definition 11 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap (c_2Eoption_2Eoption_ABS A_27a) (c_2Eoption_2Eoption_ABS A_27a))$

Definition 12 We define $c_2ErrorStateMonad_2EES_FAIL$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s \in A_27b.(c_2Eoption_2Eoption_ABS A_27a)$

Definition 13 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_2Esum_2EABS A_27a A_27b) V0e)$

Definition 14 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap (c_2Eoption_2Eoption_ABS A_27a) V0x)$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Eoption_2Eoption_CASE A_27a A_27b \in (((A_27b^{(A_27b^{A_27a})})^{A_27b})^{(ty_2Eoption_2Eoption A_27a)}) \quad (6)$$

Definition 15 We define $c_2ErrorStateMonad_2EES_CHOICE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0xM \in ((ty_2Eoption_2Eoption A_27a) A_27b)$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V0t)))) \quad (8)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (9)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A_27a.((ap V0f V2x) = (ap V1g V2x)))))) \quad (10)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0v \in A_27b.(\forall V1f \in (A_27b^{A_27a}).((ap (ap (ap (c_2Eoption_2Eoption_CASE A_27a A_27b) (c_2Eoption_2ENONE A_27a)) V0v) V1f) = V0v))) \wedge (\forall V2x \in A_27a.(\forall V3v \in A_27b.(\forall V4f \in (A_27b^{A_27a}).((ap (ap (ap (c_2Eoption_2Eoption_CASE A_27a A_27b) (ap (c_2Eoption_2ESOME A_27a) V2x)) V3v) V4f) = (ap V4f V2x)))))) \quad (11)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0xM \in ((ty_2Eoption_2Eoption\ A_27b)^{A_27a}).((ap\ (ap\ (\\ & \quad c_2ErrorStateMonad_2EES_CHOICE\ A_27b\ A_27a)\ (c_2ErrorStateMonad_2EES_FAIL \\ & \quad A_27b\ A_27a))\ V0xM) = V0xM)) \end{aligned}$$