

thm_2ErrorStateMonad_2EJOIN__MAP__JOIN (TMJieCWNpkAp8bUqKCQDFDx2G5oSy8r1fJb)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 6 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty ty_2Eone_2Eone \tag{3}$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Esum_2Esum A0 A1) \tag{4}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EABS_sum A_27a A_27b \in ((ty_2Esum_2Esum A_27a A_27b)^{((2^{A_27b})^{A_27a})^2}) \tag{5}$$

Definition 7 We define c_Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_Esum_2EABS_2EINL) V0e)$.
Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (6)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (7)$$

Definition 8 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap (c_2Eoption_2Eoption_ABS) V0x)$.

Definition 9 We define $c_2EerrorStateMonad_2EUNIT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27b.(\lambda V1s \in A_27b.V0x)$.

Definition 10 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g \in (A_27a^{A_27c}).V0f V1g$.

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b)^{(ty_2Epair_2Eprod A_27a A_27b)} \quad (8)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a)^{(ty_2Epair_2Eprod A_27a A_27b)} \quad (9)$$

Definition 11 We define $c_2Epair_2Epair_CASE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0p \in (ty_2Epair_2Epair A_27a A_27b).V0p$.

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge p x) \text{ of type } \iota \Rightarrow \iota)$.

Definition 13 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone.V0x))$.

Definition 14 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 15 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21))$.

Definition 16 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap (c_2Esum_2EABS_2EINR) V0e)$.

Definition 17 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap (c_2Eoption_2Eoption_ABS) A_27a)$.

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Eoption_2Eoption_CASE A_27a A_27b \in (((A_27b^{(A_27b^{A_27a})})^{A_27b})^{(ty_2Eoption_2Eoption A_27a)}) \quad (10)$$

Definition 18 We define $c_2EerrorStateMonad_2EBIND$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0g \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Eoption_2Eoption A_27b)})^{A_27c}.V0g$.

Definition 19 We define $c_2ErrorStateMonad_2EMMAP$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A$

Definition 20 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x)$

Definition 21 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a})$

Definition 22 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a}) A$

Definition 23 We define $c_2ErrorStateMonad_2EJOIN$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0z \in ((ty_2Eoption_2$

Assume the following.

$$True \tag{11}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V0t))) \tag{12}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{13}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{14}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A_27a.((ap V0f V2x) = (ap V1g V2x)))))) \tag{15}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c.nonempty A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27a^{A_27c}).(\forall V2x \in A_27c.((ap (ap (ap (c_2Ecombin_2Eo A_27c A_27b A_27a) V0f) V1g) V2x) = (ap V0f (ap V1g V2x)))))) \tag{16}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((ap (c_2Ecombin_2EI A_27a) V0x) = V0x)) \tag{17}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0k \in ((\\
& \quad ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27b\ A_27a))^{A_27a}). \\
& \quad (\forall V1m \in (((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27c \\
& \quad A_27a))^{A_27a})^{A_27b}). (\forall V2n \in (((ty_2Eoption_2Eoption \\
& \quad (ty_2Epair_2Eprod\ A_27d\ A_27a))^{A_27a})^{A_27c}). ((ap\ (ap\ (c_2ErrorStateMonad_2EBIND \\
& \quad A_27a\ A_27b\ A_27d)\ V0k)\ (\lambda V3a \in A_27b.(ap\ (ap\ (c_2ErrorStateMonad_2EBIND \\
& \quad A_27a\ A_27c\ A_27d)\ (ap\ V1m\ V3a))\ V2n))) = (ap\ (ap\ (c_2ErrorStateMonad_2EBIND \\
& \quad A_27a\ A_27c\ A_27d)\ (ap\ (ap\ (c_2ErrorStateMonad_2EBIND\ A_27a\ A_27b \\
& \quad A_27c)\ V0k)\ V1m))\ V2n))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0v \in A_27b. (\forall V1f \in (A_27b^{A_27a}). ((ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\
& \quad A_27a\ A_27b)\ (c_2Eoption_2ENONE\ A_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\
& \quad A_27a. (\forall V3v \in A_27b. (\forall V4f \in (A_27b^{A_27a}). ((ap\ (ap \\
& \quad (ap\ (c_2Eoption_2Eoption_CASE\ A_27a\ A_27b)\ (ap\ (c_2Eoption_2ESOME \\
& \quad A_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow (\forall V0x \in A_27b. (\forall V1y \in A_27c. (\forall V2f \in \\
& \quad ((A_27a^{A_27c})^{A_27b}). ((ap\ (ap\ (c_2Epair_2Epair_CASE\ A_27a\ A_27b \\
& \quad A_27c)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27b\ A_27c)\ V0x)\ V1y))\ V2f) = (ap \\
& \quad (ap\ V2f\ V0x)\ V1y))))))
\end{aligned} \tag{20}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad (ap\ (ap\ (c_2Ecombin_2Eo\ ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod \\ & \quad ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ ((ty_2Eoption_2Eoption \\ & \quad (ty_2Epair_2Eprod\ A_27b\ A_27a))^{A_27a})\ A_27a))^{A_27a})\ A_27a))^{A_27a}) \\ & \quad ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27b\ A_27a))^{A_27a}) \\ & \quad ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ ((ty_2Eoption_2Eoption \\ & \quad (ty_2Epair_2Eprod\ A_27b\ A_27a))^{A_27a})\ A_27a))^{A_27a}))\ (c_2ErrorStateMonad_2EJOIN \\ & \quad A_27a\ A_27b))\ (ap\ (c_2ErrorStateMonad_2EMMAP\ A_27a\ ((ty_2Eoption_2Eoption \\ & \quad (ty_2Epair_2Eprod\ A_27b\ A_27a))^{A_27a})\ ((ty_2Eoption_2Eoption \\ & \quad (ty_2Epair_2Eprod\ ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod \\ & \quad A_27b\ A_27a))^{A_27a})\ A_27a))^{A_27a}))\ (c_2ErrorStateMonad_2EJOIN \\ & \quad A_27a\ A_27b)))\ =\ (ap\ (ap\ (c_2Ecombin_2Eo\ ((ty_2Eoption_2Eoption \\ & \quad (ty_2Epair_2Eprod\ ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod \\ & \quad ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27b\ A_27a))^{A_27a}) \\ & \quad A_27a))^{A_27a})\ A_27a))^{A_27a})\ ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod \\ & \quad A_27b\ A_27a))^{A_27a})\ ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod \\ & \quad ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27b\ A_27a))^{A_27a}) \\ & \quad A_27a))^{A_27a}))\ (c_2ErrorStateMonad_2EJOIN\ A_27a\ A_27b))\ (c_2ErrorStateMonad_2EJOIN \\ & \quad A_27a\ ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27b\ A_27a))^{A_27a})))) \end{aligned}$$