thm_2EerrorStateMonad_2EMCOMP__ID (TMXRtao2Xk6sSowXz3fbXWdLMNjp2jx4YiC)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{2}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{3}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{4}$$

Definition 5 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27b}$

Let $c\_2Eoption\_2EOPTION\_\_BIND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_\_BIND\ A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b})})^{(ty\_2Eoption\_2Eoption\ A\_27} \tag{5}$$

Definition 6 *We define* c_2Eoption_2EOPTION__MCOMP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0g \in ((t$

Definition 7 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in 2$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{6}$$

Definition 9 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Definition 10 *We define* c_2Epair_2ECURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27c^{(ty\_2Epai}$

Definition 11 *We define* c_2EerrorStateMonad_2EMCOMP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda A\_27d : \iota$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{7}$$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{8}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^2)}) \tag{9}$$

Definition 12 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{10}$$

Definition 13 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Definition 14 *We define* c_2EerrorStateMonad_2EUNIT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27b.(\lambda V1s \in$

Assume the following.

$$True \tag{11}$$

Assume the following.

$$(\forall V0t \in 2.(((True\wedge(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\wedge True)\Leftrightarrow (p\ V0t))\wedge(((False\wedge(p\ V0t))\Leftrightarrow False)\wedge((((p\ V0t)\wedge False)\Leftrightarrow False)\wedge (((p\ V0t)\wedge(p\ V0t))\Leftrightarrow(p\ V0t)))))) \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow True)) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow((\mathsf{c\_2EerrorStateMonad\_2EUNIT}\ A\_27b\ A\_27a) = (ap\ (\mathsf{c\_2Epair\_2ECURRY}\ A\_27a\ A\_27b\ (ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)))\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27c. nonempty\ A\_27c\Rightarrow\forall A\_27d.nonempty\ A\_27d\Rightarrow(\forall V0g \in ((ty\_2Eoption\_2Eoption\ A\_27d)^{A\_27c}).(\forall V1f \in ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}).(((ap\ (ap\ (\mathsf{c\_2Eoption\_2EOPTION\_\_MCOMP}\ A\_27d\ A\_27c\ A\_27c)\ V0g)\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27c)) = V0g)\wedge((ap\ (ap\ (\mathsf{c\_2Eoption\_2EOPTION\_\_MCOMP}\ A\_27b\ A\_27b\ A\_27a)\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b))\ V1f) = V1f)))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27c. nonempty\ A\_27c\Rightarrow(\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}).((ap\ (\mathsf{c\_2Epair\_2ECURRY}\ A\_27a\ A\_27b\ A\_27c)\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a\ A\_27b\ A\_27c)\ V0f)) = V0f)) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27c. nonempty\ A\_27c\Rightarrow(\forall V0f \in (A\_27c^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}). ((ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a\ A\_27b\ A\_27c)\ (ap\ (\mathsf{c\_2Epair\_2ECURRY}\ A\_27a\ A\_27b\ A\_27c)\ V0f)) = V0f)) \tag{17}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow \forall A\_27e.nonempty\ A\_27e \Rightarrow \forall A\_27f.nonempty\ A\_27f \Rightarrow \forall A\_27g.nonempty\ A\_27g \Rightarrow (\forall V0g \in (((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b})^{A\_27a}).(\forall V1f \in (((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27f\ A\_27g))^{A\_27e})^{A\_27d}).(((ap\ (ap\ (\mathsf{c\_2EerrorStateMonad\_2EMCOMP}\ A\_27a\ A\_27b\ A\_27c\ A\_27a\ A\_27b)\ V0g)\ (\mathsf{c\_2EerrorStateMonad\_2EUNIT}\ A\_27b\ A\_27a)) = V0g) \wedge ((ap\ (ap\ (\mathsf{c\_2EerrorStateMonad\_2EMCOMP}\ A\_27d\ A\_27e\ (ty\_2Epair\_2Eprod\ A\_27f\ A\_27g)\ A\_27f\ A\_27g)\ (\mathsf{c\_2EerrorStateMonad\_2EUNIT}\ A\_27g\ A\_27f))\ V1f) = V1f))))$$