

thm_2ErrorStateMonad_2EMCOMP__THM
(TM-
PjHXR6G65G9Tyf9WHdkbVRGwWU7219MuJ)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_T$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ecombin_2Eo$ to be $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda A.\lambda 27c : \iota.\lambda V0f \in (A.\lambda 27b^{A-27c}).\lambda V1g$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (2)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda 27a.nonempty A.\lambda 27a \Rightarrow \forall A.\lambda 27b.nonempty A.\lambda 27b \Rightarrow c_2Epair_2ESND A.\lambda 27a A.\lambda 27b \in (A.\lambda 27b^{(ty_2Epair_2Eprod A.\lambda 27a A.\lambda 27b)}) \quad (3)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda 27a.nonempty A.\lambda 27a \Rightarrow \forall A.\lambda 27b.nonempty A.\lambda 27b \Rightarrow c_2Epair_2EFST A.\lambda 27a A.\lambda 27b \in (A.\lambda 27a^{(ty_2Epair_2Eprod A.\lambda 27a A.\lambda 27b)}) \quad (4)$$

Definition 5 We define $c_2Epair_2Epair_CASE$ to be $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda A.\lambda 27c : \iota.\lambda V0p \in (ty_2Epair_2$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty ty_2Eone_2Eone \quad (5)$$

Definition 6 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then $(the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 7 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone))$

Definition 8 We define c_2Ebool_2E2F to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 10 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21))$

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Esum_2Esum A0 A1) \quad (6)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EABS_sum A_27a A_27b \in ((ty_2Esum_2Esum A_27a A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (7)$$

Definition 12 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap (c_2Esum_2EABS_sum A_27a A_27b) V0e)$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (8)$$

Definition 13 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap (c_2Eoption_2Eoption_ABS A_27a) (c_2Eone_2Eone))$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Eoption_2Eoption_CASE A_27a A_27b \in (((A_27b)^{(A_27b^{A_27a})})^{A_27b})^{(ty_2Eoption_2Eoption A_27a)} \quad (9)$$

Definition 14 We define $c_2EerrorStateMonad_2EBIND$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0g \in ((ty_2Eoption_2Eoption A_27a) \rightarrow A_27b) \rightarrow A_27c$

Definition 15 We define $c_2EerrorStateMonad_2EEXT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0g \in (((ty_2Eoption_2Eoption A_27a) \rightarrow A_27b) \rightarrow A_27c) \rightarrow A_27c$

Definition 16 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c)^{A_27b}) \rightarrow A_27a$

Let $c_2Eoption_2EOPTION_BIND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Eoption_2EOPTION_BIND A_27a A_27b \in (((ty_2Eoption_2Eoption A_27a)^{(ty_2Eoption_2Eoption A_27a)^{A_27b}})^{(ty_2Eoption_2Eoption A_27a)}) \quad (10)$$

Definition 17 We define $c_2Eoption_2EOPTION_MCOMP$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0g \in ($
 Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (11)$$

Definition 18 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2E$

Definition 19 We define $c_2Epair_2ECURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27c^{(ty_2Epair$

Definition 20 We define $c_2ErrorStateMonad_2EMCOMP$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27d : \iota$

Definition 21 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 22 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 23 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS$

Definition 24 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption_2E$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (13)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).((V0f = \\ V1g) \Leftrightarrow (\forall V2x \in A_27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27a^{A_27c}). \\ (\forall V2x \in A_27c.((ap\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27c\ A_27b\ A_27a) \\ V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption \\ A_27a).((V0opt = (c_2Eoption_2ENONE\ A_27a)) \vee (\exists V1x \in A_27a. \\ (V0opt = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0v \in A_27b. (\forall V1f \in (A_27b^{A_27a}). ((ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\
& A_27a\ A_27b)\ (c_2Eoption_2ENONE\ A_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\
& A_27a. (\forall V3v \in A_27b. (\forall V4f \in (A_27b^{A_27a}). ((ap\ (ap \\
& (ap\ (c_2Eoption_2Eoption_CASE\ A_27a\ A_27b)\ (ap\ (c_2Eoption_2ESOME \\
& A_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0f \in ((ty_2Eoption_2Eoption\ A_27a)^{A_27b}). ((ap\ (ap\ (\\
& c_2Eoption_2EOPTION_BIND\ A_27a\ A_27b)\ (c_2Eoption_2ENONE\ A_27b)) \\
& V0f) = (c_2Eoption_2ENONE\ A_27a))) \wedge (\forall V1x \in A_27b. (\forall V2f \in \\
& ((ty_2Eoption_2Eoption\ A_27a)^{A_27b}). ((ap\ (ap\ (c_2Eoption_2EOPTION_BIND \\
& A_27a\ A_27b)\ (ap\ (c_2Eoption_2ESOME\ A_27b)\ V1x))\ V2f) = (ap\ V2f\ V1x))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow (\forall V0f \in ((A_27c^{A_27b})^{A_27a}). (\forall V1v \in \\
& (ty_2Epair_2Eprod\ A_27a\ A_27b). ((ap\ (ap\ (c_2Epair_2EUNCURRY \\
& A_27a\ A_27b\ A_27c)\ V0f)\ V1v) = (ap\ (ap\ V0f\ (ap\ (c_2Epair_2EFST\ A_27a \\
& A_27b)\ V1v))\ (ap\ (c_2Epair_2ESND\ A_27a\ A_27b)\ V1v))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow (\forall V0f \in ((A_27c^{A_27b})^{A_27a}). (\forall V1x \in \\
& A_27a. (\forall V2y \in A_27b. ((ap\ (ap\ (c_2Epair_2EUNCURRY\ A_27a \\
& A_27b\ A_27c)\ V0f)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V1x)\ V2y)) = \\
& (ap\ (ap\ V0f\ V1x)\ V2y))))))
\end{aligned} \tag{20}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0g \in ((\\
& (ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27c\ A_27b))^{A_27b})^{A_27d}). \\
& (\forall V1f \in (((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27d \\
& A_27b))^{A_27b})^{A_27a}). ((ap\ (ap\ (c_2ErrorStateMonad_2EMCOMP \\
& A_27a\ A_27b\ (ty_2Epair_2Eprod\ A_27c\ A_27b)\ A_27d\ A_27b)\ V0g)\ V1f) = \\
& (ap\ (ap\ (c_2Ecombin_2Eo\ A_27a\ ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod \\
& A_27c\ A_27b))^{A_27b})\ ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod \\
& A_27d\ A_27b))^{A_27b}))\ (ap\ (c_2ErrorStateMonad_2EEXT\ A_27b\ A_27c \\
& A_27d)\ V0g))\ V1f))))))
\end{aligned}$$