

thm_2ErrorStateMonad_2EMMAP__COMP
(TM-
SkXX41ogBhqAZNSSodsJPcxHYDXvAEPaY)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow q Q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (2)$$

Definition 6 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty ty_2Eone_2Eone \quad (3)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Esum_2Esum A0 A1) \quad (4)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum \\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \end{aligned} \quad (5)$$

Definition 7 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS_sum\ V0e))$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (6)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in \\ ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \end{aligned} \quad (7)$$

Definition 8 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption_ABS\ V0x))$

Definition 9 We define $c_2ErrorStateMonad_2EUNIT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27b.(\lambda V1s \in A_27a.V0x)$

Definition 10 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g \in (A_27a^{A_27c}).V0f\ V1g$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (8)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (9)$$

Definition 11 We define $c_2Epair_2Epair_CASE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0p \in (ty_2Epair_2Epair\ A_27a\ A_27b).V0p$

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A).P\ x)$ of type $\iota \Rightarrow \iota$.

Definition 13 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone))\ (\lambda V0x \in ty_2Eone_2Eone.V0x)$

Definition 14 We define $c_2Ebool_2E_21$ to be $(ap\ (c_2Ebool_2E_21\ 2))\ (\lambda V0t \in 2.V0t)$.

Definition 15 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_21))$

Definition 16 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS_sum\ V0e))$

Definition 17 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ (\lambda V0x \in A_27a.V0x))$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2Eoption_CASE \\ & A_27a\ A_27b \in (((A_27b^{(A_27b^{A_27a})})^{A_27b})^{(ty_2Eoption_2Eoption\ A_27a)}) \end{aligned} \quad (10)$$

Definition 18 We define $c_2ErrorStateMonad_2EBIND$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0g \in ((ty_$

Definition 19 We define $c_2ErrorStateMonad_2EMMAP$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ & A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\ & True)) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ & A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).((V0f = \\ & V1g) \Leftrightarrow (\forall V2x \in A_27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27c^{A_27b}). \\ & (\forall V2x \in A_27c.((ap\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27c\ A_27b\ A_27a) \\ & V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0k \in ((\\ & ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27b\ A_27a))^{A_27a}). \\ & (\forall V1m \in (((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27c \\ & A_27a))^{A_27a})^{A_27b}).(\forall V2n \in (((ty_2Eoption_2Eoption \\ & (ty_2Epair_2Eprod\ A_27d\ A_27a))^{A_27a})^{A_27c}).((ap\ (ap\ (c_2ErrorStateMonad_2EBIND \\ & A_27a\ A_27b\ A_27d)\ V0k)\ (\lambda V3a \in A_27b.(ap\ (ap\ (c_2ErrorStateMonad_2EBIND \\ & A_27a\ A_27c\ A_27d)\ (ap\ V1m\ V3a))\ V2n))) = (ap\ (ap\ (c_2ErrorStateMonad_2EBIND \\ & A_27a\ A_27c\ A_27d)\ (ap\ (ap\ (c_2ErrorStateMonad_2EBIND\ A_27a\ A_27b \\ & A_27c)\ V0k)\ V1m))\ V2n)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0v \in A_27b. (\forall V1f \in (A_27b^{A_27a}). ((ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\
& A_27a\ A_27b)\ (c_2Eoption_2ENONE\ A_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\
& A_27a. (\forall V3v \in A_27b. (\forall V4f \in (A_27b^{A_27a}). ((ap\ (ap \\
& (ap\ (c_2Eoption_2Eoption_CASE\ A_27a\ A_27b)\ (ap\ (c_2Eoption_2ESOME \\
& A_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow (\forall V0x \in A_27b. (\forall V1y \in A_27c. (\forall V2f \in \\
& ((A_27a^{A_27c})^{A_27b}). ((ap\ (ap\ (c_2Epair_2Epair_CASE\ A_27a\ A_27b \\
& A_27c)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27b\ A_27c)\ V0x)\ V1y))\ V2f) = (ap \\
& (ap\ V2f\ V0x)\ V1y))))))
\end{aligned} \tag{19}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0f \in (A_27b^{A_27a}). \\
& (\forall V1g \in (A_27a^{A_27c}). ((ap\ (c_2ErrorStateMonad_2EMMAP \\
& A_27d\ A_27b\ A_27c)\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27c\ A_27b\ A_27a)\ V0f) \\
& V1g)) = (ap\ (ap\ (c_2Ecombin_2Eo\ ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod \\
& A_27c\ A_27d))^{A_27d})\ ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod \\
& A_27b\ A_27d))^{A_27d})\ ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod \\
& A_27a\ A_27d))^{A_27d}))\ (ap\ (c_2ErrorStateMonad_2EMMAP\ A_27d\ A_27b \\
& A_27a)\ V0f))\ (ap\ (c_2ErrorStateMonad_2EMMAP\ A_27d\ A_27a\ A_27c) \\
& V1g))))))
\end{aligned}$$