thm_2EerrorStateMonad_2EMMAP__JOIN (TMGyWvog15x9arXeTiRnk7WRAgBZ3uhhjqs)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{2}$$

Definition 6 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{3}$$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{4}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^2)}) \tag{5}$$

Definition 7 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS\_$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (6)$$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (7)$$

Definition 8 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_A$

Definition 9 *We define* c_2EerrorStateMonad_2EUNIT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27b.(\lambda V1s \in A$

Definition 10 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (8)$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (9)$$

Definition 11 *We define* c_2Epair_2Epair__CASE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0p \in (ty\_2Epair\_$

Definition 12 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 13 *We define* c_2Eone_2Eone *to be* $(ap\ ($c_2Emin_2E_40 $ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Definition 14 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Definition 15 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ $c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 16 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Definition 17 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Let $c\_2Eoption\_2Eoption\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \quad (10)$$

Definition 18 *We define* c_2EerrorStateMonad_2EBIND *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0g \in ((ty\_$

Definition 19 *We define* c_2EerrorStateMonad_2EMMAP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A$

Definition 20 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Definition 21 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Definition 22 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap\ (\mathsf{c\_2Ecombin\_2ES}\ A\_27a\ (A\_27a^{A\_27a})\ A$

Definition 23 *We define* c_2EerrorStateMonad_2EJOIN *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0z \in ((ty\_2Eoption\_2$

Assume the following.

$$True \tag{11}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27b^{A\_27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A\_27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27a^{A\_27c}).(\forall V2x \in A\_27c.((ap\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27c\ A\_27b\ A\_27a)\ V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0k \in ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27a))^{A\_27a}).(\forall V1m \in (((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27c\ A\_27a))^{A\_27a})^{A\_27b}).(\forall V2n \in (((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27d\ A\_27a))^{A\_27a})^{A\_27c}).((ap\ (ap\ (\mathsf{c\_2EerrorStateMonad\_2EBIND}\ A\_27a\ A\_27b\ A\_27d)\ V0k)\ (\lambda V3a \in A\_27b.(ap\ (ap\ (\mathsf{c\_2EerrorStateMonad\_2EBIND}\ A\_27a\ A\_27c\ A\_27d)\ (ap\ V1m\ V3a))\ V2n))) = (ap\ (ap\ (\mathsf{c\_2EerrorStateMonad\_2EBIND}\ A\_27a\ A\_27c\ A\_27d)\ (ap\ (ap\ (\mathsf{c\_2EerrorStateMonad\_2EBIND}\ A\_27a\ A\_27b\ A\_27c)\ V0k)\ V1m))\ V2n))))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0v \in A\_27b.(\forall V1f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ V0v)\ V1f) = V0v))) \land (\forall V2x \in A\_27a.(\forall V3v \in A\_27b.(\forall V4f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0x \in A\_27b.(\forall V1y \in A\_27c.(\forall V2f \in ((A\_27a^{A\_27c})^{A\_27b}).((ap\ (ap\ (\mathsf{c\_2Epair\_2Epair\_\_CASE}\ A\_27a\ A\_27b\ A\_27c)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27b\ A\_27c)\ V0x)\ V1y))\ V2f) = (ap\ (ap\ V2f\ V0x)\ V1y))))) \tag{20}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27c))^{A\_27c})\ A\_27c))^{A\_27c})\ ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27c))^{A\_27c})\ ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27c))^{A\_27c}))\ (ap\ (\mathsf{c\_2EerrorStateMonad\_2EMMAP}\ A\_27c\ A\_27b\ A\_27a)\ V0f))\ (\mathsf{c\_2EerrorStateMonad\_2EJOIN}\ A\_27c\ A\_27a)) = (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27c))^{A\_27c})\ A\_27c))^{A\_27c})\ ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27c))^{A\_27c})\ ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27c))^{A\_27c})\ A\_27c))^{A\_27c}))\ (\mathsf{c\_2EerrorStateMonad\_2EJOIN}\ A\_27c\ A\_27b))\ (ap\ (\mathsf{c\_2EerrorStateMonad\_2EMMAP}\ A\_27c\ ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27c))^{A\_27c})\ ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27c))^{A\_27c}))\ (ap\ (\mathsf{c\_2EerrorStateMonad\_2EMMAP}\ A\_27c\ A\_27b\ A\_27a)\ V0f)))))$$