

# thm\_2Eextreal\_2ECROSS\_COUNTABLE (TMHS9UPyYw3s8p6MRK2w3kJDbwQhbis2uFM)

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**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A$ .if  $(\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ ).

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ecombin\_2EC$  to be  $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda A.\lambda 27c : \iota.(\lambda V0f \in ((A.\lambda 27c^{A.\lambda 27b})^{A.\lambda 27a}))$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A.\lambda 27a})).(ap (ap (c\_2Emin\_2E\_3D (2^{A.\lambda 27a})))$

**Definition 6** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda A.\lambda 27c : \iota.\lambda V0f \in (A.\lambda 27b^{A.\lambda 27c}).\lambda V1g$

**Definition 7** We define  $c\_2Emarker\_2EAbbrev$  to be  $\lambda V0x \in 2.V0x$ .

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.\lambda 27a.nonempty A.\lambda 27a \Rightarrow \forall A.\lambda 27b.nonempty A.\lambda 27b \Rightarrow c\_2Epair\_2ESND A.\lambda 27a A.\lambda 27b \in (A.\lambda 27b^{(ty\_2Epair\_2Eprod A.\lambda 27a A.\lambda 27b)}) \tag{2}$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.\lambda 27a.nonempty A.\lambda 27a \Rightarrow \forall A.\lambda 27b.nonempty A.\lambda 27b \Rightarrow c\_2Epair\_2EFST A.\lambda 27a A.\lambda 27b \in (A.\lambda 27a^{(ty\_2Epair\_2Eprod A.\lambda 27a A.\lambda 27b)}) \tag{3}$$

**Definition 8** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda A.\lambda 27c : \iota.\lambda V0f \in ((A.\lambda 27c^{A.\lambda 27b})^{A.\lambda 27a})$

**Definition 9** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A.\lambda 27a})).(ap V0P (ap (c\_2Emin\_2E\_40 A.\lambda 27a)))$

**Definition 10** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. (\lambda V1f \in (2^{A\_27a}). (ap\ V1f\ V0x)))$

**Definition 11** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 12** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2. (ap\ V2t\ t1\ t2))))$

**Definition 13** We define  $c\_2Epred\_set\_2ESURJ$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A\_27a}). \lambda V1s \in (2^{A\_27b})$

**Definition 14** We define  $c\_2Epred\_set\_2EINJ$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A\_27a}). \lambda V1s \in (2^{A\_27b})$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (4)$$

**Definition 15** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap\ (c\_2Epair\_2EABS\_prod\ x\ y))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}}) \end{aligned} \quad (5)$$

**Definition 16** We define  $c\_2Epred\_set\_2ECROSS$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0P \in (2^{A\_27a}). \lambda V1Q \in (2^{A\_27b})$

**Definition 17** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A\_27a}). \lambda V1s \in (2^{A\_27b})$

**Definition 18** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. c\_2Ebool\_2E2ET)$ .

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (6)$$

**Definition 19** We define  $c\_2Epred\_set\_2EBIJ$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A\_27a}). \lambda V1s \in (2^{A\_27b})$

**Definition 20** We define  $c\_2Epred\_set\_2Eenumerate$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). (ap\ (c\_2Emin\_2E\_40\ V0s))$

**Definition 21** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2. (ap\ V2t\ t1\ t2))))$

**Definition 22** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. \lambda V1s \in (2^{A\_27a}). (ap\ (c\_2Emin\_2E\_40\ V0x\ s))$

**Definition 23** We define  $c\_2Ebool\_2E2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2. V0t))$ .

**Definition 24** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. c\_2Ebool\_2E2EF)$ .

**Definition 25** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). (ap\ (c\_2Ebool\_2E\_21\ 2)\ V0s)$

**Definition 26** We define  $c\_2Epred\_set\_2Ecountable$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). (ap\ (c\_2Ebool\_2E\_3F\ V0s))$

**Definition 27** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_7E$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (9)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (12)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (13)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (14)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (16)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B) \wedge (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C))))))) \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (18)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_{.27} \in 2. (\forall V2y \in 2. (\forall V3y_{.27} \in 2. (((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (19)$$

Assume the following.

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0a \in A_{.27a}. (\exists V1x \in A_{.27a}. (V1x = V0a))) \quad (20)$$

Assume the following.

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}). (\forall V1a \in A_{.27a}. ((\exists V2x \in A_{.27a}. ((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (21)$$

Assume the following.

$$(p (ap (c\_2Epred\_set\_2Ecountable (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum))) (ap (ap (c\_2Epred\_set\_2ECROSS ty\_2Enum\_2Enum ty\_2Enum\_2Enum) (c\_2Epred\_set\_2EUNIV ty\_2Enum\_2Enum))) (c\_2Epred\_set\_2EUNIV ty\_2Enum\_2Enum)))) \quad (22)$$

Assume the following.

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow \forall A_{.27b}. \text{nonempty } A_{.27b} \Rightarrow (\forall V0t \in (2^{A_{.27b}}). (\forall V1s \in (2^{A_{.27a}}). (((p (ap (c\_2Epred\_set\_2Ecountable A_{.27a}) V1s)) \wedge ((p (ap (c\_2Epred\_set\_2EFINITE A_{.27a}) V1s)) \wedge (p (ap (c\_2Epred\_set\_2Ecountable A_{.27b}) V0t)))) \Rightarrow (p (ap (c\_2Epred\_set\_2Ecountable (ty\_2Epair\_2Eprod A_{.27a} A_{.27b})) (ap (ap (c\_2Epred\_set\_2ECROSS A_{.27a} A_{.27b}) V1s) V0t)))))) \quad (23)$$

Assume the following.

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow \forall A_{.27b}. \text{nonempty } A_{.27b} \Rightarrow (\forall V0t \in (2^{A_{.27b}}). (\forall V1s \in (2^{A_{.27a}}). (((p (ap (c\_2Epred\_set\_2Ecountable A_{.27a}) V1s)) \wedge ((p (ap (c\_2Epred\_set\_2Ecountable A_{.27b}) V0t)) \wedge (p (ap (c\_2Epred\_set\_2EFINITE A_{.27b}) V0t)))) \Rightarrow (p (ap (c\_2Epred\_set\_2Ecountable (ty\_2Epair\_2Eprod A_{.27a} A_{.27b})) (ap (ap (c\_2Epred\_set\_2ECROSS A_{.27a} A_{.27b}) V1s) V0t)))))) \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow ( \\ & \quad \forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.(\forall V2a \in A_{.27a}.(\forall V3b \in \\ & \quad A_{.27b}.(((ap\ (ap\ (c_{.2Epair\_2E\_2C}\ A_{.27a}\ A_{.27b})\ V0x)\ V1y) = (ap\ (ap \\ & \quad (c_{.2Epair\_2E\_2C}\ A_{.27a}\ A_{.27b})\ V2a)\ V3b))) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \\ & \hspace{15em} (25) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow ( \\ & \quad \forall V0x \in (ty_{.2Epair\_2Eprod}\ A_{.27a}\ A_{.27b}).(\exists V1q \in A_{.27a}. \\ & \quad (\exists V2r \in A_{.27b}.(V0x = (ap\ (ap\ (c_{.2Epair\_2E\_2C}\ A_{.27a}\ A_{.27b}) \\ & \quad V1q)\ V2r)))))) \\ & \hspace{15em} (26) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow ( \\ & \quad \forall V0x \in (ty_{.2Epair\_2Eprod}\ A_{.27a}\ A_{.27b}).((ap\ (ap\ (c_{.2Epair\_2E\_2C} \\ & \quad A_{.27a}\ A_{.27b})\ (ap\ (c_{.2Epair\_2EFST}\ A_{.27a}\ A_{.27b})\ V0x))\ (ap\ (c_{.2Epair\_2ESND} \\ & \quad A_{.27a}\ A_{.27b})\ V0x)) = V0x)) \\ & \hspace{15em} (27) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow ( \\ & \quad \forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.((ap\ (c_{.2Epair\_2EFST}\ A_{.27a} \\ & \quad A_{.27b})\ (ap\ (ap\ (c_{.2Epair\_2E\_2C}\ A_{.27a}\ A_{.27b})\ V0x)\ V1y)) = V0x))) \\ & \hspace{15em} (28) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow ( \\ & \quad \forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.((ap\ (c_{.2Epair\_2ESND}\ A_{.27a} \\ & \quad A_{.27b})\ (ap\ (ap\ (c_{.2Epair\_2E\_2C}\ A_{.27a}\ A_{.27b})\ V0x)\ V1y)) = V1y))) \\ & \hspace{15em} (29) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow \forall A_{.27c}. \\ & \quad nonempty\ A_{.27c} \Rightarrow (\forall V0f \in ((A_{.27c}^{A_{.27b}})^{A_{.27a}}).(\forall V1x \in \\ & \quad A_{.27a}.(\forall V2y \in A_{.27b}.((ap\ (ap\ (c_{.2Epair\_2EUNCURRY}\ A_{.27a} \\ & \quad A_{.27b}\ A_{.27c})\ V0f)\ (ap\ (ap\ (c_{.2Epair\_2E\_2C}\ A_{.27a}\ A_{.27b})\ V1x)\ V2y)) = \\ & \quad (ap\ (ap\ V0f\ V1x)\ V2y)))))) \\ & \hspace{15em} (30) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in \\ & \quad (2^{A_{.27a}}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_{.27a}.((p\ (ap\ (ap\ (c_{.2Ebool\_2EIN} \\ & \quad A_{.27a})\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_{.2Ebool\_2EIN}\ A_{.27a})\ V2x)\ V1t)))))) \\ & \hspace{15em} (31) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}).(\forall V1v \in \\ & A\_27a.((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V1v)\ (ap\ (c\_2Epred\_set\_2EGSPEC \\ & \quad A\_27a\ A\_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A\_27b.((ap\ (ap\ (c\_2Epair\_2E\_2C \\ & \quad A\_27a\ 2)\ V1v)\ c\_2Ebool\_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ (c\_2Epred\_set\_2EUNIV\ A\_27a)))) \quad (33)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0f \in (A\_27b^{A\_27a}).(\forall V1s \in (2^{A\_27a}).((p\ (ap\ (c\_2Epred\_set\_2Ecountable \\ & \quad A\_27a)\ V1s)) \Rightarrow (p\ (ap\ (c\_2Epred\_set\_2Ecountable\ A\_27b)\ (ap\ (ap \\ & \quad (c\_2Epred\_set\_2EIMAGE\ A\_27a\ A\_27b)\ V0f)\ V1s)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).((p\ (ap \\ & (c\_2Epred\_set\_2Ecountable\ A\_27a)\ V0s)) \Leftrightarrow ((p\ (ap\ (c\_2Epred\_set\_2EFINITE \\ & \quad A\_27a)\ V0s)) \vee (p\ (ap\ (ap\ (ap\ (c\_2Epred\_set\_2EBIJ\ ty\_2Enum\_2Enum \\ & \quad A\_27a)\ (ap\ (c\_2Epred\_set\_2Eenumerate\ A\_27a)\ V0s))\ (c\_2Epred\_set\_2EUNIV \\ & \quad ty\_2Enum\_2Enum))\ V0s)))))) \end{aligned} \quad (35)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (40)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg( \\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (( \\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{45}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{46}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{47}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{48}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{49}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{50}$$

**Theorem 1**

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow \forall A_{27b}. \text{nonempty } A_{27b} \Rightarrow ( \\ & \forall V_0 t \in (2^{A_{27b}}). (\forall V_1 s \in (2^{A_{27a}}). (((p (ap (c\_2Epred\_set\_2Ecountable \\ & A_{27a}) V_1 s)) \wedge (p (ap (c\_2Epred\_set\_2Ecountable A_{27b}) V_0 t))) \Rightarrow \\ & (p (ap (c\_2Epred\_set\_2Ecountable (ty\_2Epair\_2Eprod A_{27a} A_{27b})) \\ & (ap (ap (c\_2Epred\_set\_2ECROSS A_{27a} A_{27b}) V_1 s) V_0 t)))))) \end{aligned}$$