

thm_2Eextreal_2EEXTREAL_SUM_IMAGE_EXTREAL_SUM_
(TMEsURsTDe-
vuMZKTU81qKEreJW6VBsrGJWg)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2EBOUNDED$ to be $(\lambda V0v \in 2.c_2Ebool_2ET)$.

Definition 4 We define $c_2Ecombin_2ES$ to be $\lambda A.\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 5 We define $c_2Ecombin_2EK$ to be $\lambda A.\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 6 We define $c_2Ecombin_2EI$ to be $\lambda A.\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a})) A_27a))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 7 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \tag{4}$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (5)$$

Let $ty_2Eextreal_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eextreal_2Eextreal \quad (6)$$

Let $c_2Eextreal_2ENormal : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENormal \in (ty_2Eextreal_2Eextreal^{ty_2Erealax_2Ereal}) \quad (7)$$

Definition 8 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 9 We define $c_2Eextreal_2Eextreal_of_num$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap c_2Eextreal_2Eextreal_of_num$

Definition 10 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1g \in (A_27c^{A_27a}).$

Let $c_2Eextreal_2Eextreal_add : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_add \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \quad (8)$$

Let $c_2Epred_set_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epred_set_2EITSET \\ A_27a\ A_27b \in (((A_27b^{A_27a})^{(2^{A_27a})})^{((A_27b^{A_27a})^{A_27a})}) \end{aligned} \quad (9)$$

Definition 11 We define $c_2Eextreal_2EEXTREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota. \lambda V0f \in (ty_2Eextreal_2Eextreal)$

Definition 12 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 13 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (c_2Ebool_2E_21\ 2)\ V2t))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (10)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (11)$$

Definition 14 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epair_2E_2C\ V0x\ V1y))$

Definition 15 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (17)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (21)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (22)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (V0x = V0x)) \quad (23)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (24)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (26)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A-27a}).(\neg(\exists V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B))))))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A) \vee (p V1B)))))) \quad (31)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (32)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in 2.(((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \quad (33)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A-27a}).(\forall V1a \in A.27a.((\exists V2x \in A.27a.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (34)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0f \in (2^{A-27a}).(\forall V1v \in A.27a.((\forall V2x \in A.27a.((V2x = V1v) \Rightarrow (p (ap V0f V2x)))) \Leftrightarrow (p (ap V0f V1v)))))) \quad (35)$$

Assume the following.

$$(\forall V0v \in 2.((p (ap c.2Ebool_2EBOUNDED V0v)) \Leftrightarrow True)) \quad (36)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((ap (c.2Ecombin_2EI A_27a) V0x) = V0x)) \quad (37)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \forall V0f \in (A_27b^{A_27a}).(((ap (ap (c.2Ecombin_2Eo A_27a A_27b \\ & A_27b) (c.2Ecombin_2EI A_27b)) V0f) = V0f) \wedge ((ap (ap (c.2Ecombin_2Eo \\ & A_27a A_27b A_27a) V0f) (c.2Ecombin_2EI A_27a)) = V0f))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0f \in (ty_2Eextreal_2Eextreal^{A_27a}). \\ & (((ap (ap (c.2Eextreal_2EEXTREAL_SUM_IMAGE A_27a) V0f) (c.2Epred_set_2EEMPTY \\ & A_27a)) = (ap c.2Eextreal_2Eextreal_of_num c.2Enum_2E0)) \wedge \\ & (\forall V1e \in A_27a.(\forall V2s \in (2^{A_27a}).((p (ap (c.2Epred_set_2EFINITE \\ & A_27a) V2s)) \Rightarrow ((ap (ap (c.2Eextreal_2EEXTREAL_SUM_IMAGE A_27a) \\ & V0f) (ap (ap (c.2Epred_set_2EINSERT A_27a) V1e) V2s)) = (ap (ap \\ & c.2Eextreal_2Eextreal_add (ap V0f V1e)) (ap (ap (c.2Eextreal_2EEXTREAL_SUM_IMAGE \\ & A_27a) V0f) (ap (ap (c.2Epred_set_2EDELETE A_27a) V2s) V1e)))))))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \forall V0s \in (2^{A_27a}).((p (ap (c.2Epred_set_2EFINITE A_27a) \\ & V0s)) \Rightarrow (\forall V1f_27 \in (A_27b^{A_27a}).((p (ap (ap (ap (c.2Epred_set_2EINJ \\ & A_27a A_27b) V1f_27) V0s) (ap (ap (c.2Epred_set_2EIMAGE A_27a \\ & A_27b) V1f_27) V0s))) \Rightarrow (\forall V2f \in (ty_2Eextreal_2Eextreal^{A_27b}). \\ & ((ap (ap (c.2Eextreal_2EEXTREAL_SUM_IMAGE A_27b) V2f) (ap (\\ & ap (c.2Epred_set_2EIMAGE A_27a A_27b) V1f_27) V0s)) = (ap (ap (\\ & c.2Eextreal_2EEXTREAL_SUM_IMAGE A_27a) (ap (ap (c.2Ecombin_2Eo \\ & A_27a ty_2Eextreal_2Eextreal A_27b) V2f) V1f_27)) V0s)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1s_27 \in \\ & (2^{A_27a}).(((p (ap (c.2Epred_set_2EFINITE A_27a) V0s)) \wedge ((p \\ & (ap (c.2Epred_set_2EFINITE A_27a) V1s_27)) \wedge (p (ap (ap (c.2Epred_set_2EDISJOINT \\ & A_27a) V0s) V1s_27)))) \Rightarrow (\forall V2f \in (ty_2Eextreal_2Eextreal^{A_27a}). \\ & ((ap (ap (c.2Eextreal_2EEXTREAL_SUM_IMAGE A_27a) V2f) (ap (\\ & ap (c.2Epred_set_2EUNION A_27a) V0s) V1s_27)) = (ap (ap c.2Eextreal_2Eextreal_add \\ & (ap (ap (c.2Eextreal_2EEXTREAL_SUM_IMAGE A_27a) V2f) V0s)) \\ & (ap (ap (c.2Eextreal_2EEXTREAL_SUM_IMAGE A_27a) V2f) V1s_27)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\ & \quad A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\ & \quad (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in (ty_2Epair_2Eprod\ A_27a\ A_27b). (\exists V1q \in A_27a. \\ & \quad (\exists V2r \in A_27b. (V0x = (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ \\ & \quad V1q)\ V2r)))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. ((ap\ (c_2Epair_2EFST\ A_27a \\ & \quad A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V0x))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. ((ap\ (c_2Epair_2ESND\ A_27a \\ & \quad A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V1y))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & \quad (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & \quad A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg (p\ (ap\ (ap \\ & \quad (c_2Ebool_2EIN\ A_27a)\ V0x)\ (c_2Epred_set_2EEMPTY\ A_27a)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & \quad (2^{A_27a}). (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\ & \quad V2x)\ (ap\ (ap\ (c_2Epred_set_2EUNION\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\ & \quad (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & \quad A_27a)\ V2x)\ V1t)))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & \quad (2^{A_27a}). (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\ & \quad V2x)\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\ & \quad (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & \quad A_27a)\ V2x)\ V1t)))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in \\ A_{.27a}.(\forall V2s \in (2^{A_{.27a}}).((p\ (ap\ (ap\ (c_{.2Ebool_2EIN}\ A_{.27a}) \\ V0x)\ (ap\ (ap\ (c_{.2Epred_set_2EINSERT}\ A_{.27a})\ V1y)\ V2s))) \Leftrightarrow ((V0x = \\ V1y) \vee (p\ (ap\ (ap\ (c_{.2Ebool_2EIN}\ A_{.27a})\ V0x)\ V2s)))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1s \in \\ (2^{A_{.27a}}).((\neg(p\ (ap\ (ap\ (c_{.2Ebool_2EIN}\ A_{.27a})\ V0x)\ V1s))) \Leftrightarrow ((ap \\ (c_{.2Epred_set_2EDELETE}\ A_{.27a})\ V1s)\ V0x) = V1s)))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\ \forall V0y \in A_{.27b}.(\forall V1s \in (2^{A_{.27a}}).(\forall V2f \in (A_{.27b}^{A_{.27a}}). \\ ((p\ (ap\ (ap\ (c_{.2Ebool_2EIN}\ A_{.27b})\ V0y)\ (ap\ (ap\ (c_{.2Epred_set_2EIMAGE} \\ A_{.27a}\ A_{.27b})\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A_{.27a}.((V0y = (ap\ V2f\ V3x)) \wedge \\ (p\ (ap\ (ap\ (c_{.2Ebool_2EIN}\ A_{.27a})\ V3x)\ V1s)))))) \end{aligned} \quad (52)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0P \in (2^{(2^{A_{.27a}})}).((\\ (p\ (ap\ V0P\ (c_{.2Epred_set_2EEMPTY}\ A_{.27a}))) \wedge (\forall V1s \in (2^{A_{.27a}}). \\ (((p\ (ap\ (c_{.2Epred_set_2EFINITE}\ A_{.27a})\ V1s)) \wedge (p\ (ap\ V0P\ V1s))) \Rightarrow \\ (\forall V2e \in A_{.27a}.((\neg(p\ (ap\ (ap\ (c_{.2Ebool_2EIN}\ A_{.27a})\ V2e)\ V1s))) \Rightarrow \\ (p\ (ap\ V0P\ (ap\ (ap\ (c_{.2Epred_set_2EINSERT}\ A_{.27a})\ V2e)\ V1s)))))) \Rightarrow \\ (\forall V3s \in (2^{A_{.27a}}).((p\ (ap\ (c_{.2Epred_set_2EFINITE}\ A_{.27a}) \\ V3s)) \Rightarrow (p\ (ap\ V0P\ V3s)))))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\ \forall V0s \in (2^{A_{.27a}}).((p\ (ap\ (c_{.2Epred_set_2EFINITE}\ A_{.27a}) \\ V0s)) \Rightarrow (\forall V1f \in (A_{.27b}^{A_{.27a}}).((p\ (ap\ (c_{.2Epred_set_2EFINITE} \\ A_{.27b})\ (ap\ (ap\ (c_{.2Epred_set_2EIMAGE}\ A_{.27a}\ A_{.27b})\ V1f)\ V0s)))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\ \forall V0P \in (2^{A_{.27a}}).(\forall V1Q \in (2^{A_{.27b}}).(\forall V2x \in \\ (ty_2Epair_2Eprod\ A_{.27a}\ A_{.27b}).((p\ (ap\ (ap\ (c_{.2Ebool_2EIN}\ (ty_2Epair_2Eprod \\ A_{.27a}\ A_{.27b})\ V2x)\ (ap\ (ap\ (c_{.2Epred_set_2ECROSS}\ A_{.27a}\ A_{.27b}) \\ V0P)\ V1Q))) \Leftrightarrow ((p\ (ap\ (ap\ (c_{.2Ebool_2EIN}\ A_{.27a})\ (ap\ (c_{.2Epair_2EFST} \\ A_{.27a}\ A_{.27b})\ V2x))\ V0P)) \wedge (p\ (ap\ (ap\ (c_{.2Ebool_2EIN}\ A_{.27b})\ (ap\ (c_{.2Epair_2ESND} \\ A_{.27a}\ A_{.27b})\ V2x))\ V1Q)))))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow \forall A_{.27c}. \\
& nonempty\ A_{.27c} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(((ap\ (ap\ (c_{.2}Epred_set_2ECROSS \\
& A_{.27a}\ A_{.27b})\ V0P)\ (c_{.2}Epred_set_2EEMPTY\ A_{.27b})) = (c_{.2}Epred_set_2EEMPTY \\
& (ty_{.2}Epair_{.2}Eprod\ A_{.27a}\ A_{.27b}))) \wedge ((ap\ (ap\ (c_{.2}Epred_set_2ECROSS \\
& A_{.27c}\ A_{.27a})\ (c_{.2}Epred_set_2EEMPTY\ A_{.27c}))\ V0P) = (c_{.2}Epred_set_2EEMPTY \\
& (ty_{.2}Epair_{.2}Eprod\ A_{.27c}\ A_{.27a}))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\
& \forall V0P \in (2^{A_{.27a}}).(\forall V1Q \in (2^{A_{.27b}}).(((p\ (ap\ (c_{.2}Epred_set_2EFINITE \\
& A_{.27a})\ V0P)) \wedge (p\ (ap\ (c_{.2}Epred_set_2EFINITE\ A_{.27b})\ V1Q))) \Rightarrow (p \\
& (ap\ (c_{.2}Epred_set_2EFINITE\ (ty_{.2}Epair_{.2}Eprod\ A_{.27a}\ A_{.27b})) \\
& (ap\ (ap\ (c_{.2}Epred_set_2ECROSS\ A_{.27a}\ A_{.27b})\ V0P)\ V1Q))))))
\end{aligned} \tag{57}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{58}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))
\end{aligned} \tag{61}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee (\neg(\\
& p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\
& (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p))))))))
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee \neg(p \ V1q)) \wedge (((p \ V0p) \vee \neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee \neg(p \ V0p))))))))))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee \neg(p \ V2r))) \wedge (\\
& \neg(p \ V1q) \vee ((p \ V2r) \vee \neg(p \ V0p))))))))))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow \neg(p \ V1q)) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge (\neg(p \ V1q) \vee \neg(p \ V0p))))))
\end{aligned} \tag{67}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \ V0p) \Rightarrow (p \ V1q)) \Rightarrow (p \ V0p))) \tag{68}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \ V0p) \Rightarrow (p \ V1q)) \Rightarrow \neg(p \ V1q))) \tag{69}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \ V0p) \vee (p \ V1q)) \Rightarrow \neg(p \ V0p))) \tag{70}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \ V0p) \vee (p \ V1q)) \Rightarrow \neg(p \ V1q))) \tag{71}$$

Assume the following.

$$(\forall V0p \in 2. (\neg(\neg(p \ V0p)) \Rightarrow (p \ V0p))) \tag{72}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow (\\
& \quad \forall V0s \in (2^{A.27a}). (\forall V1s.27 \in (2^{A.27b}). (\forall V2f \in \\
& ((ty_2Eextreal_2Eextreal^{A.27b})^{A.27a}). (((p \ (ap \ (c_2Epred_set_2EFINITE \\
& \quad A.27a) \ V0s)) \wedge (p \ (ap \ (c_2Epred_set_2EFINITE \ A.27b) \ V1s.27))) \Rightarrow \\
& ((ap \ (ap \ (c_2Eextreal_2EEXTREAL_SUM_IMAGE \ A.27a) \ (\lambda V3x \in \\
& \quad A.27a. (ap \ (ap \ (c_2Eextreal_2EEXTREAL_SUM_IMAGE \ A.27b) \ (ap \\
& \quad V2f \ V3x)) \ V1s.27))) \ V0s) = (ap \ (ap \ (c_2Eextreal_2EEXTREAL_SUM_IMAGE \\
& \quad (ty_2Epair_2Eprod \ A.27a \ A.27b)) \ (\lambda V4x \in (ty_2Epair_2Eprod \\
& \quad A.27a \ A.27b). (ap \ (ap \ V2f \ (ap \ (c_2Epair_2EFST \ A.27a \ A.27b) \ V4x)) \\
& (ap \ (c_2Epair_2ESND \ A.27a \ A.27b) \ V4x)))) \ (ap \ (ap \ (c_2Epred_set_2ECROSS \\
& \quad A.27a \ A.27b) \ V0s) \ V1s.27))))))
\end{aligned}$$