

thm_2Eextreal_2EEXTREAL__SUM__IMAGE__IMAGE
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Jbsf3TstkVU2nkY9Qp97G9FqEgHA)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))))$

Definition 5 We define $c_2Ecombin_2E_o$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.\lambda V0f \in (A.27b^{A-27c}).\lambda V1g$

Let $ty_2Eextreal_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eextreal_2Eextreal \tag{1}$$

Let $c_2Eextreal_2Eextreal_add : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_add \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \tag{2}$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{3}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{4}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{5}$$

Definition 6 We define c_2Enum_2E0 to be (ap $c_2Enum_2EABS_num$ $c_2Enum_2EZERO_REP$).

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (6)$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_2Eextreal_2ENormal : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENormal \in (ty_2Eextreal_2Eextreal^{ty_2Erealax_2Ereal}) \quad (8)$$

Definition 7 We define $c_2Eextreal_2Eextreal_of_num$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ c_2Eextreal_2Eextreal_of_num\ n)$.

Let $c_2Epred_set_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EITSET \\ A_27a\ A_27b \in (((A_27b^{A_27b})^{(2^{A_27a})})^{((A_27b^{A_27b})^{A_27a})}) \end{aligned} \quad (9)$$

Definition 8 We define $c_2Eextreal_2EEXTREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Eextreal_2Eextreal\ A_27a)$.

Definition 9 We define c_2Ebool_2EF to be (ap (c_2Ebool_2E21 2) ($\lambda V0t \in 2.V0t$)).

Definition 10 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 11 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$.

Definition 12 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 13 We define $c_2Ebool_2E5C_2F$ to be ($\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2.V2t))$).

Definition 14 We define $c_2Ebool_2E2F_5C$ to be ($\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2.V2t))$).

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod \\ A0\ A1) \end{aligned} \quad (10)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (11)$$

Definition 15 We define c_2Epair_2E2C to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2Epair\ x\ y))$.

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \end{aligned} \quad (12)$$

Definition 16 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap\ (c_2E$

Definition 17 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ (c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E$

Definition 18 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2E$

Definition 19 We define $c_2Epred_set_2EDELETE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1x \in A_27a.(ap\ (a$

Definition 20 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 21 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^A$

Definition 22 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 23 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap\ (c_2Ebool_2E_21\ (2$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee \neg(p\ V0t))) \quad (16)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\
& (p \ V0t) \Rightarrow False) \Leftrightarrow \neg(p \ V0t))))))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{21}$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(V0x = V0x)) \tag{22}$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \tag{23}$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p \ V0t))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\neg(\exists V1x \in A.27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p \ (ap \ V0P \ V2x)))))) \tag{26}$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in 2.(((\forall V2x \in A.27a.(p \ (ap \ V0P \ V2x))) \wedge (p \ V1Q)) \Leftrightarrow (\forall V3x \in A.27a.((p \ (ap \ V0P \ V3x)) \wedge (p \ V1Q)))))) \tag{27}$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A.27a}).((\forall V2x \in A.27a.((p \ (ap \ V1P \ V2x)) \vee (p \ V0Q))) \Leftrightarrow ((\forall V3x \in A.27a.(p \ (ap \ V1P \ V3x))) \vee (p \ V0Q)))))) \tag{28}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A_27a. (p\ (ap\ V1Q\ V3x)))))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee (\neg(p\ V1B)))))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \wedge (\neg(p\ V1B))))))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee ((p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C)))))) \quad (31)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (32)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c.nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27c^{A_27a}). (\forall V2x \in A_27c. ((ap\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27c\ A_27b\ A_27a)\ V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in (ty_2Eextreal_2Eextreal^{A_27a}). (((ap\ (ap\ (c_2Eextreal_2EEXTREAL_SUM_IMAGE\ A_27a)\ V0f)\ (c_2Epred_set_2EEMPTY\ A_27a)) = (ap\ c_2Eextreal_2Eextreal_of_num\ c_2Enum_2E0)) \wedge ((\forall V1e \in A_27a. (\forall V2s \in (2^{A_27a}). ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ V2s)) \Rightarrow ((ap\ (ap\ (c_2Eextreal_2EEXTREAL_SUM_IMAGE\ A_27a)\ V0f)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V1e)\ V2s)) = (ap\ (ap\ c_2Eextreal_2Eextreal_add\ (ap\ V0f\ V1e))\ (ap\ (ap\ (c_2Eextreal_2EEXTREAL_SUM_IMAGE\ A_27a)\ V0f)\ (ap\ (ap\ (c_2Epred_set_2EDELETE\ A_27a)\ V2s)\ V1e)))))))))) \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. (\forall V2s \in (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V1y)\ V2s))) \Leftrightarrow ((V0x = V1y) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ V2s)))))) \quad (36)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1s \in \\ & (2^{A_27a}). ((\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ V1s))) \Leftrightarrow ((ap \\ & (ap\ (c_2Epred_set_2EDELETE\ A_27a)\ V1s)\ V0x) = V1s)))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b^{A_27a}). \\ & ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\ & A_27a\ A_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\ & (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in (A_27b^{A_27a}). ((ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_27a \\ & A_27b)\ V0f)\ (c_2Epred_set_2EEMPTY\ A_27a)) = (c_2Epred_set_2EEMPTY \\ & A_27b))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in (A_27b^{A_27a}). (\forall V1x \in A_27a. (\forall V2s \in (\\ & 2^{A_27a}). ((ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_27a\ A_27b)\ V0f)\ (ap \\ & (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V1x)\ V2s)) = (ap\ (ap\ (c_2Epred_set_2EINSERT \\ & A_27b)\ (ap\ V0f\ V1x))\ (ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_27a\ A_27b) \\ & V0f)\ V2s)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(2^{A_27a})}). ((\\ & (p\ (ap\ V0P\ (c_2Epred_set_2EEMPTY\ A_27a))) \wedge (\forall V1s \in (2^{A_27a}). \\ & ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ V1s)) \wedge (p\ (ap\ V0P\ V1s))) \Rightarrow \\ & (\forall V2e \in A_27a. ((\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2e)\ V1s))) \Rightarrow \\ & (p\ (ap\ V0P\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V2e)\ V1s)))))) \Rightarrow \\ & (\forall V3s \in (2^{A_27a}). ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a) \\ & V3s)) \Rightarrow (p\ (ap\ V0P\ V3s)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0s \in (2^{A_27a}). ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a) \\ & V0s)) \Rightarrow (\forall V1f \in (A_27b^{A_27a}). (p\ (ap\ (c_2Epred_set_2EFINITE \\ & A_27b)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_27a\ A_27b)\ V1f)\ V0s)))))) \end{aligned} \quad (42)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (44)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (45)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (46)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (51)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (52)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (53)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (54)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (57)$$

Theorem 1

$$\begin{aligned} & \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow \forall A_{.27b}. \text{nonempty } A_{.27b} \Rightarrow (\\ & \forall V0s \in (2^{A_{.27a}}). ((p (ap (c_{.2} \text{Epred_set_2EFINITE } A_{.27a}) \\ & V0s)) \Rightarrow (\forall V1f_{.27} \in (A_{.27b}^{A_{.27a}}). ((p (ap (ap (ap (c_{.2} \text{Epred_set_2EINJ} \\ & A_{.27a} A_{.27b}) V1f_{.27}) V0s) (ap (ap (c_{.2} \text{Epred_set_2EIMAGE } A_{.27a} \\ & A_{.27b}) V1f_{.27}) V0s))) \Rightarrow (\forall V2f \in (ty_{.2} \text{Eextreal_2Eextreal}^{A_{.27b}}). \\ & ((ap (ap (c_{.2} \text{Eextreal_2EEXTREAL_SUM_IMAGE } A_{.27b}) V2f) (ap (\\ & ap (c_{.2} \text{Epred_set_2EIMAGE } A_{.27a} A_{.27b}) V1f_{.27}) V0s)) = (ap (ap (\\ & c_{.2} \text{Eextreal_2EEXTREAL_SUM_IMAGE } A_{.27a}) (ap (ap (c_{.2} \text{Ecombin_2Eo} \\ & A_{.27a} ty_{.2} \text{Eextreal_2Eextreal } A_{.27b}) V2f) V1f_{.27}) V0s))))))))) \end{aligned}$$