

thm\_2Eextreal\_2EEXTREAL\_\_SUM\_\_IMAGE\_\_POS\_\_MEM\_\_LE  
 (TMcVoVbWwqAh-  
 mDn9phUbUpNF7ccGU5s79Em)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ if  $(\exists x \in A.p (ap P x))$  **then** (the  $(\lambda x.x \in A \wedge p)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A$

**Definition 4** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x$

Let  $ty\_2Eextreal\_2Eextreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eextreal\_2Eextreal \tag{1}$$

Let  $c\_2Eextreal\_2Eextreal\_add : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_add \in ((ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \tag{2}$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in omega \tag{3}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{4}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{5}$$

**Definition 5** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealx\_2Ereal \quad (6)$$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_2Eextreal\_2ENormal : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2ENormal \in (ty\_2Eextreal\_2Eextreal^{ty\_2Erealx\_2Ereal}) \quad (8)$$

**Definition 6** We define  $c\_2Ebool\_2E21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (c\_2Emin\_2E3D (2^{A\_27a})))$

**Definition 7** We define  $c\_2Eextreal\_2Eextreal\_of\_num$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap c\_2Eextreal\_2Eextreal\_of\_num)$

Let  $c\_2Epred\_set\_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EITSET \\ A\_27a\ A\_27b \in (((A\_27b^{A\_27b})^{(2^{A\_27a})})^{((A\_27b^{A\_27b})^{A\_27a})}) \end{aligned} \quad (9)$$

**Definition 8** We define  $c\_2Eextreal\_2EEXTREAL\_SUM\_IMAGE$  to be  $\lambda A\_27a : \iota. \lambda V0f \in (ty\_2Eextreal\_2Eextreal)$

Let  $c\_2Eextreal\_2Eextreal\_le : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_le \in ((2^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \quad (10)$$

**Definition 9** We define  $c\_2Ebool\_2E2F$  to be  $(ap (c\_2Ebool\_2E21\ 2)) (\lambda V0t \in 2.V0t)$ .

**Definition 10** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. c\_2Ebool\_2E2F)$ .

**Definition 11** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. (\lambda V1f \in (2^{A\_27a}). (ap\ V1f\ V0x)))$

**Definition 12** We define  $c\_2Emin\_2E3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 13** We define  $c\_2Ebool\_2E5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E21\ 2)) (\lambda V2t \in 2. (c\_2Ebool\_2E21\ V2t))))$

**Definition 14** We define  $c\_2Ebool\_2E2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E21\ 2)) (\lambda V2t \in 2. (c\_2Ebool\_2E21\ V2t))))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (11)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (12)$$

**Definition 15** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b})}) \end{aligned} \quad (13)$$

**Definition 16** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_7E$

**Definition 17** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_7E$

**Definition 18** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_7E$

**Definition 19** We define  $c\_2Epred\_set\_2EDELETE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1x \in A\_27a.(ap (c\_2Ebool\_2E\_7E$

**Definition 20** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_7E$

Assume the following.

$$True \quad (14)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \end{aligned} \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (16)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} ((\forall V0t \in 2.((\neg (\neg (p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (20)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \quad (21)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\ & p\ V0t)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in A\_27a.(p\ (ap\ V1P\ V3x)) \vee (p\ V0Q)))))) \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ( \\ & (p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in \\ & 2.(((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Eextreal\_2Eextreal.(p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_le \\ & V0x)\ V0x))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0w \in ty\_2Eextreal\_2Eextreal.(\forall V1x \in ty\_2Eextreal\_2Eextreal. \\ & (\forall V2y \in ty\_2Eextreal\_2Eextreal.(\forall V3z \in ty\_2Eextreal\_2Eextreal. \\ & (((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_le\ V0w)\ V1x)) \wedge (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_le \\ & V2y)\ V3z))) \Rightarrow (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_le\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_add \\ & V0w)\ V2y))\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_add\ V1x)\ V3z)))))) \end{aligned} \quad (29)$$

Assume the following.

$$(\forall V0x \in ty\_2Eextreal\_2Eextreal.((ap (ap c\_2Eextreal\_2Eextreal\_add V0x) (ap c\_2Eextreal\_2Eextreal\_of\_num c\_2Enum\_2E0)) = V0x)) \quad (30)$$

Assume the following.

$$(\forall V0x \in ty\_2Eextreal\_2Eextreal.((ap (ap c\_2Eextreal\_2Eextreal\_add (ap c\_2Eextreal\_2Eextreal\_of\_num c\_2Enum\_2E0)) V0x) = V0x)) \quad (31)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}). \\ & (((ap (ap (c\_2Eextreal\_2EEXTREAL\_SUM\_IMAGE A\_27a) V0f) (c\_2Epred\_set\_2EEMPTY \\ & A\_27a)) = (ap c\_2Eextreal\_2Eextreal\_of\_num c\_2Enum\_2E0)) \wedge \\ & (\forall V1e \in A\_27a. (\forall V2s \in (2^{A\_27a}). ((p (ap (c\_2Epred\_set\_2EFINITE \\ & A\_27a) V2s)) \Rightarrow ((ap (ap (c\_2Eextreal\_2EEXTREAL\_SUM\_IMAGE A\_27a) \\ & V0f) (ap (ap (c\_2Epred\_set\_2EINSERT A\_27a) V1e) V2s)) = (ap (ap \\ & c\_2Eextreal\_2Eextreal\_add (ap V0f V1e)) (ap (ap (c\_2Eextreal\_2EEXTREAL\_SUM\_IMAGE \\ & A\_27a) V0f) (ap (ap (c\_2Epred\_set\_2EDELETE A\_27a) V2s) V1e)))))))))) \quad (32) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}). \\ & (\forall V1s \in (2^{A\_27a}). (((p (ap (c\_2Epred\_set\_2EFINITE A\_27a) \\ & V1s)) \wedge (\forall V2x \in A\_27a. ((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V2x) \\ & V1s)) \Rightarrow (p (ap (ap c\_2Eextreal\_2Eextreal\_le (ap c\_2Eextreal\_2Eextreal\_of\_num \\ & c\_2Enum\_2E0)) (ap V0f V2x)))))) \Rightarrow (p (ap (ap c\_2Eextreal\_2Eextreal\_le \\ & (ap c\_2Eextreal\_2Eextreal\_of\_num c\_2Enum\_2E0)) (ap (ap (c\_2Eextreal\_2EEXTREAL\_SUM\_IMAGE \\ & A\_27a) V0f) V1s)))))) \quad (33) \end{aligned}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. (\neg (p (ap (ap (c\_2Ebool\_2EIN A\_27a) V0x) (c\_2Epred\_set\_2EEMPTY A\_27a)))))) \quad (34)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in \\ & A\_27a. (\forall V2s \in (2^{A\_27a}). ((p (ap (ap (c\_2Ebool\_2EIN A\_27a) \\ & V0x) (ap (ap (c\_2Epred\_set\_2EINSERT A\_27a) V1y) V2s))) \Leftrightarrow ((V0x = \\ & V1y) \vee (p (ap (ap (c\_2Ebool\_2EIN A\_27a) V0x) V2s)))))) \quad (35) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1s \in \\ & (2^{A\_27a}). ((\neg (p (ap (ap (c\_2Ebool\_2EIN A\_27a) V0x) V1s))) \Leftrightarrow ((ap \\ & (ap (c\_2Epred\_set\_2EDELETE A\_27a) V1s) V0x) = V1s)))) \quad (36) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in (2^{(2^{A\_27a})}). (( \\
& \quad (p (ap V0P (c\_2Epred\_set\_2EEMPTY A\_27a))) \wedge (\forall V1s \in (2^{A\_27a}). \\
& \quad ((p (ap (c\_2Epred\_set\_2EFINITE A\_27a) V1s)) \wedge (p (ap V0P V1s))) \Rightarrow \\
& \quad (\forall V2e \in A\_27a. ((\neg (p (ap (ap (c\_2Ebool\_2EIN A\_27a) V2e) V1s))) \Rightarrow \\
& \quad (p (ap V0P (ap (ap (c\_2Epred\_set\_2EINSERT A\_27a) V2e) V1s)))))) \Rightarrow \\
& \quad (\forall V3s \in (2^{A\_27a}). ((p (ap (c\_2Epred\_set\_2EFINITE A\_27a) \\
& \quad V3s)) \Rightarrow (p (ap V0P V3s))))))
\end{aligned} \tag{37}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{38}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow \\
& \quad ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False}))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg((p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow \\
& \quad ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False}))))))
\end{aligned} \tag{41}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& \quad (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg( \\
& \quad p V2r)) \vee (\neg(p V1q))) \wedge ((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& \quad ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& \quad (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& \quad (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& \quad (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& \quad ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\neg(p V1q) \vee ((p V2r) \vee \neg(p V0p)))))))))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (\neg(p V1q) \vee \neg(p V0p)))))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow \neg(p V1q)))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V0p)))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V1q)))) \quad (51)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (52)$$

**Theorem 1**

$$\forall A. \forall 27a. \text{nonempty } A. 27a \Rightarrow (\forall V0f \in (ty. 2Eextreal. 2Eextreal^{A-27a}). (\forall V1s \in (2^{A-27a}). (((p (ap (c. 2Epred\_set. 2EFINITE A. 27a) V1s)) \wedge (\forall V2x \in A. 27a. ((p (ap (ap (c. 2Ebool. 2EIN A. 27a) V2x) V1s)) \Rightarrow (p (ap (ap c. 2Eextreal. 2Eextreal\_le (ap c. 2Eextreal. 2Eextreal\_of\_num c. 2Enum. 2E0)) (ap V0f V2x)))))) \Rightarrow (\forall V3x \in A. 27a. ((p (ap (ap (c. 2Ebool. 2EIN A. 27a) V3x) V1s)) \Rightarrow (p (ap (ap c. 2Eextreal. 2Eextreal\_le (ap V0f V3x)) (ap (ap (c. 2Eextreal. 2EEXTREAL\_SUM\_IMAGE A. 27a) V0f) V1s))))))))))$$