

thm_2Eextreal_2Ediv__one
(TMQu5dQ7dVfJz5CSB3xCyYsdzTmkmtsY4AW)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Eextreal_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eextreal_2Eextreal \tag{1}$$

Let $c_2Eextreal_2Eextreal_inv : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_inv \in (ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal}) \tag{2}$$

Let $c_2Eextreal_2Eextreal_mul : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_mul \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \tag{3}$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 4 We define $c_2Eextreal_2Eextreal_div$ to be $\lambda V0x \in ty_2Eextreal_2Eextreal.\lambda V1y \in ty_2Eextreal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \tag{4}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{5}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \tag{6}$$

Definition 5 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 6 We define $c_2\text{Earithmetic_EZERO}$ to be $c_2\text{Enum_E0}$.

Let $c_2\text{Enum_EREP_num} : \iota$ be given. Assume the following.

$$c_2\text{Enum_EREP_num} \in (\omega^{ty_2\text{Enum_Enum}}) \quad (7)$$

Let $c_2\text{Enum_ESUC_REP} : \iota$ be given. Assume the following.

$$c_2\text{Enum_ESUC_REP} \in (\omega^{\omega}) \quad (8)$$

Definition 7 We define $c_2\text{Enum_ESUC}$ to be $\lambda V0m \in ty_2\text{Enum_Enum} . (ap\ c_2\text{Enum_EABS_num} ($

Let $c_2\text{Earithmetic_E_2B} : \iota$ be given. Assume the following.

$$c_2\text{Earithmetic_E_2B} \in ((ty_2\text{Enum_Enum}^{ty_2\text{Enum_Enum}})^{ty_2\text{Enum_Enum}}) \quad (9)$$

Definition 8 We define $c_2\text{Earithmetic_EBIT1}$ to be $\lambda V0n \in ty_2\text{Enum_Enum} . (ap\ (ap\ c_2\text{Earithmetic_E_2B} ($

Definition 9 We define $c_2\text{Earithmetic_ENUMERAL}$ to be $\lambda V0x \in ty_2\text{Enum_Enum} . V0x$.

Let $ty_2\text{Erealax_2Ereal} : \iota$ be given. Assume the following.

$$nonempty\ ty_2\text{Erealax_2Ereal} \quad (10)$$

Let $c_2\text{Ereal_2Ereal_of_num} : \iota$ be given. Assume the following.

$$c_2\text{Ereal_2Ereal_of_num} \in (ty_2\text{Erealax_2Ereal}^{ty_2\text{Enum_Enum}}) \quad (11)$$

Let $c_2\text{Eextreal_2ENormal} : \iota$ be given. Assume the following.

$$c_2\text{Eextreal_2ENormal} \in (ty_2\text{Eextreal_2Eextreal}^{ty_2\text{Erealax_2Ereal}}) \quad (12)$$

Definition 10 We define $c_2\text{Eextreal_2Eextreal_of_num}$ to be $\lambda V0n \in ty_2\text{Enum_Enum} . (ap\ c_2\text{Eextreal_2Eextreal_of_num} ($

Assume the following.

$$True \quad (13)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a . ((V0x = V0x) \Leftrightarrow True)) \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2\text{Eextreal_2Eextreal} . ((ap\ (ap\ c_2\text{Eextreal_2Eextreal_mul} \\ & V0x) (ap\ c_2\text{Eextreal_2Eextreal_of_num} (ap\ c_2\text{Earithmetic_ENUMERAL} \\ & (ap\ c_2\text{Earithmetic_EBIT1}\ c_2\text{Earithmetic_EZERO)))) = V0x)) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & ((ap\ c_2\text{Eextreal_2Eextreal_inv} (ap\ c_2\text{Eextreal_2Eextreal_of_num} \\ & (ap\ c_2\text{Earithmetic_ENUMERAL} (ap\ c_2\text{Earithmetic_EBIT1}\ c_2\text{Earithmetic_EZERO)))) = \\ & (ap\ c_2\text{Eextreal_2Eextreal_of_num} (ap\ c_2\text{Earithmetic_ENUMERAL} \\ & (ap\ c_2\text{Earithmetic_EBIT1}\ c_2\text{Earithmetic_EZERO)))) \end{aligned} \quad (16)$$

Theorem 1

$(\forall V0x \in ty_2Eextreal_2Eextreal.((ap (ap c_2Eextreal_2Eextreal_div$
 $V0x) (ap c_2Eextreal_2Eextreal_of_num (ap c_2Earithmetic_2ENUMERAL$
 $(ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) = V0x))$