

thm\_2Eextreal\_2Einf\_suc (TMFGN-nvKssgL4bJjY8WEcy4wLpG8HAEG4qN)

October 26, 2020

Let  $ty\_2Eextreal\_2Eextreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eextreal\_2Eextreal \quad (1)$$

Let  $c\_2Eextreal\_2Eextreal\_le : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_le \in ((2^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \quad (2)$$

Let  $c\_2Eextreal\_2Eextreal\_ainv : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_ainv \in (ty\_2Eextreal\_2Eextreal)^{ty\_2Eextreal\_2Eextreal} \quad (3)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. (\lambda V1f \in (2^{A\_27a}). (ap V1f V0x)))$

**Definition 3** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p \ P \Rightarrow p \ Q)$  of type  $\iota$ .

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) (\lambda V1t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2. inj\_o (V1t1 = V2t)))))))$

**Definition 6** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2. inj\_o (V1t2 = V2t)))))))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod \\ A0\ A1) \end{aligned} \quad (4)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (5)$$

**Definition 7** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap (c\_2Epair\_2Eprod ty\_2Epair\_2Eprod A\_27a A\_27b) (ty\_2Epair\_2Eprod A\_27a A\_27b)))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty A\_27a \Rightarrow \forall A\_27b. nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a A\_27b \in ((2^{A\_27a})((ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b})) \end{aligned} \quad (6)$$

**Definition 8** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A\_27a}). \lambda V1s \in (A\_27a^{A\_27b})$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty ty\_2Erealax\_2Ereal \quad (7)$$

Let  $c\_2Eextreal\_2ENormal : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2ENormal \in (ty\_2Eextreal\_2Eextreal^{ty\_2Erealax\_2Ereal}) \quad (8)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty ty\_2Ehreal\_2Ehreal \quad (9)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (10)$$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota))$

**Definition 10** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal. (ap (c\_2Emin\_2E\_40 (ty\_2Epair\_2Eprod ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal))) V0a)$

Let  $c\_2Erealax\_2Etreal\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_lt \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal)}) \quad (11)$$

**Definition 11** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal. \lambda V1T2 \in ty\_2Erealax\_2Ereal. (ap (c\_2Erealax\_2Etreal\_lt (ty\_2Epair\_2Eprod ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal))) V0T1 V1T2)$

**Definition 12** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap V0P (ap (c\_2Emin\_2E\_40 (ty\_2Epair\_2Eprod ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal))) V0P)))$

**Definition 13** We define  $c\_2Ereal\_2Esup$  to be  $\lambda V0P \in (2^{ty\_2Erealax\_2Ereal}). (ap (c\_2Emin\_2E\_40 ty\_2Erealax\_2Ereal)))$

Let  $c\_2Eextreal\_2ENegInf : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2ENegInf \in ty\_2Eextreal\_2Eextreal \quad (12)$$

**Definition 14** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2)) (\lambda V0t \in 2. V0t)$ .

**Definition 15** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. (ap (c\_2Ebool\_2Eprod ty\_2Epair\_2Eprod A\_27a A\_27b))) V1t1 V2t2)))$

Let  $c_2Eextreal\_2EPosInf : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2EPosInf \in ty\_2Eextreal\_2Eextreal \quad (13)$$

**Definition 16** We define  $c\_2Eextreal\_2Eextreal\_sup$  to be  $\lambda V0p \in (2^{ty\_2Eextreal\_2Eextreal}).(ap\ (ap\ (ap\ (c\_2Eextreal\_2Eextreal\ p)\ 0)\ V)\ 0)$ .

**Definition 17** We define  $c_{\text{2Eextreal\_inf}}$  to be  $\lambda V0p \in (2^{ty\_2Eextreal\_2Eextreal}).(ap\ c_{\text{2Eextreal\_inf}})$ .

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

*nonempty* *ty\_2Enum\_2Enum* (14)

Let  $c_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (15)$$

Let  $c_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (16)$$

**Definition 18** We define  $c_2Eb0o_2E_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Eb0o_2E_7E))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (17)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^\omega) \quad (18)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (19)$$

**Definition 19** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum$

**Definition 20** We define  $c_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 21** We define c\_2Earithmetic\_2E\_3E to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 22** We define  $c_{\text{Ebool}} : \mathcal{F}$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c_{\text{Ebool}}_21_2)(\lambda V2t \in$

**Definition 23** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

*c\_2Enum\_2ZERO\_REPO*  $\in \omega$

define c\_2Enum\_2E0 to be (ap c\_2Enum\_2EABS\_num c\_2E

Let  $c_2$  be given. Assume the following.

$$c_2Earithmetic\_2EXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (21)$$

Let  $c_2Earithmetic_2E_2D : \iota$  be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (22)$$

Let  $c_2$  be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum}^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (23)$$

**Definition 26** We define c\_2Earthmetic\_2ENUMERAL to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

**Definition 27** We define  $c\_2Enumeral\_2EiSUC$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2ESUC\ (ap$

Let  $c_2$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \\ (24)$$

**Definition 28** We define  $c\_2E\text{numeral}\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

**Definition 29** We define  $c\_2Earthmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earthmetic\ 2EBIT2\ n)\ V)$

**Definition 30** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap (ap c\_2Earithmetic$

**Definition 31** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

**Definition 32** We define  $c\_2Earthmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum.\lambda$

**Definition 33** We define  $\text{c\_2Epred\_set\_2EUNIV}$  to be  $\lambda A.\_27a : \iota.(\lambda V0x \in A.\_27a.c\_2Ebool\_2ET)$ .

Assume the following.

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B V1n) V0m)))) \quad (26)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
 & \forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2B V0m) \\
 & (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p)) = (ap (ap c\_2Earithmetic\_2E\_2B \\
 & (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) V2p)))))) \\
 \end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
 & ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge \\
 & (((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\
 & (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL \\
 & (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge \\
 & (((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL \\
 & (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge ( \\
 & ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m) V1n) = (ap \\
 & (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) \\
 & V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = \\
 & (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A \\
 & V0m) V1n)))))))))) \\
 \end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
 & \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) (ap (ap c\_2Earithmetic\_2E\_2B \\
 & V0m) V2p))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p)))))) \\
 \end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
 & (\neg(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & (ap c\_2Enum\_2ESUC V1n)) V0m)))))) \\
 \end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. ((ap c\_2Enum\_2ESUC V0n) = (ap (ap \\
 & c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
 & c\_2Earithmetic\_2EZERO))) V0n))) \\
 \end{aligned} \tag{31}$$

Assume the following.

$$True \tag{32}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
 V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \tag{33}$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_{27a}. (p V0t)) \Leftrightarrow (p V0t))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (36)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (37)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0x \in A_{27a}. (V0x = V0x)) \quad (38)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0x \in A_{27a}. (\forall V1y \in A_{27a}. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (39)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (40)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (41)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (42)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_{27} \in 2. (\forall V2y \in 2. (\forall V3y_{27} \in 2. (((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27}))))))) \quad (43)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty\_2Eextreal\_2Eextreal. (\forall V1y \in ty\_2Eextreal\_2Eextreal. \\
 & (\forall V2z \in ty\_2Eextreal\_2Eextreal. (((p (ap (ap c\_2Eextreal\_2Eextreal\_le \\
 & V0x) V1y)) \wedge (p (ap (ap c\_2Eextreal\_2Eextreal\_le V1y) V2z))) \Rightarrow \\
 & p (ap (ap c\_2Eextreal\_2Eextreal\_le V0x) V2z)))))) \\
 \end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in (2^{ty\_2Eextreal\_2Eextreal}). (\forall V1x \in ty\_2Eextreal\_2Eextreal. \\
 & ((p (ap (ap c\_2Eextreal\_2Eextreal\_le V1x) (ap c\_2Eextreal\_2Eextreal\_inf \\
 & V0p))) \Leftrightarrow (\forall V2y \in ty\_2Eextreal\_2Eextreal. ((p (ap V0p V2y)) \Rightarrow \\
 & (p (ap (ap c\_2Eextreal\_2Eextreal\_le V1x) V2y))))))) \\
 \end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in (2^{ty\_2Eextreal\_2Eextreal}). (\forall V1x \in ty\_2Eextreal\_2Eextreal. \\
 & ((p (ap (ap c\_2Eextreal\_2Eextreal\_le (ap c\_2Eextreal\_2Eextreal\_inf \\
 & V0p)) V1x)) \Leftrightarrow (\forall V2y \in ty\_2Eextreal\_2Eextreal. ((\forall V3z \in \\
 & ty\_2Eextreal\_2Eextreal. ((p (ap V0p V3z)) \Rightarrow (p (ap (ap c\_2Eextreal\_2Eextreal\_le \\
 & V2y) V3z)))) \Rightarrow (p (ap (ap c\_2Eextreal\_2Eextreal\_le V2y) V1x))))))) \\
 \end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in (2^{ty\_2Eextreal\_2Eextreal}). (\forall V1x \in ty\_2Eextreal\_2Eextreal. \\
 & (((ap c\_2Eextreal\_2Eextreal\_inf V0p) = V1x) \Leftrightarrow ((\forall V2y \in ty\_2Eextreal\_2Eextreal. \\
 & ((p (ap V0p V2y)) \Rightarrow (p (ap (ap c\_2Eextreal\_2Eextreal\_le V1x) V2y)))) \wedge \\
 & (\forall V3y \in ty\_2Eextreal\_2Eextreal. ((\forall V4z \in ty\_2Eextreal\_2Eextreal. \\
 & ((p (ap V0p V4z)) \Rightarrow (p (ap (ap c\_2Eextreal\_2Eextreal\_le V3y) V4z)))) \Rightarrow \\
 & (p (ap (ap c\_2Eextreal\_2Eextreal\_le V3y) V1x))))))) \\
 \end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0n) = V0n)) \wedge (\forall V1n \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge (\forall V2n \in ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2E_2B V3m) = ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2E0) V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2E0) V3m))) \wedge (\forall V4n \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& (\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2E_2A V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge (\forall V6n \in ty\_2Enum\_2Enum.(\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL V6n) V7m))) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2E0) V7m))) \wedge (\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2E_2D c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge (\forall V9n \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge (\forall V10n \in ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2E_2D (ap c_2Earithmetic_2ENUMERAL V10n) V11m))) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V10n) V11m)))) \wedge (\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2EEEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V12n)))) = c_2Enum_2E0)) \wedge (\forall V13n \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2EEEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 V13n)))) = c_2Enum_2E0)) \wedge (\forall V14n \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2EEEXP V14n) c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \wedge \\
& (\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2EEEXP (ap c_2Earithmetic_2ENUMERAL V15n)) (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2EEEXP V15n) V16m)))) \wedge (((ap c_2Enum_2ESUC c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \wedge (\forall V17n \in ty\_2Enum\_2Enum.((ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2ESUC V17n)))) \wedge (((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = c_2Enum_2E0) \wedge (\forall V18n \in ty\_2Enum\_2Enum.((ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Eprim_rec_2EPRE V18n)))) \wedge (\forall V19n \in ty\_2Enum\_2Enum.((((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge (\forall V20n \in ty\_2Enum\_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge (\forall V21n \in ty\_2Enum\_2Enum.((\forall V22m \in ty\_2Enum\_2Enum.(((ap c_2Earithmetic_2ENUMERAL V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge (\forall V24n \in ty\_2Enum\_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V24n)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) V24n)))) \wedge (\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL V25n)) (ap c_2Earithmetic_2ENUMERAL V26m)))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V25n) V26m)))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge (\forall V28n \in ty\_2Enum\_2Enum.((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V28n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3E c_2Earithmetic_2EZERO) V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V29n)) \Leftrightarrow True)) \wedge (\forall V30m \in ty\_2Enum\_2Enum.((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V30m)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3D c_2Earithmetic_2EZERO) V30m)))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge (\forall V32n \in ty\_2Enum\_2Enum.((p (ap (ap c_2Earithmetic_2E_3D c_2Earithmetic_2ENUMERAL V32n)) \Leftrightarrow True)))
\end{aligned}$$

Assume the following.

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Earithmetic\_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap c\_2Earithmetic\_2EBIT2 V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V1m) V0n))) \wedge ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))))))))))) \\
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1x \in \\
& A\_27a. ((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V1x) V0P)) \Leftrightarrow (p (ap V0P V1x)))))) \\
\end{aligned} \tag{51}$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. (p (ap (ap (c\_2Ebool\_2EIN \\
A\_27a) V0x) (c\_2Epred\_set\_2EUNIV A\_27a)))) \tag{52}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow ( \\
& \forall V0y \in A\_27b. (\forall V1s \in (2^{A\_27a}). (\forall V2f \in (A\_27b)^{A\_27a}. \\
& ((p (ap (ap (c\_2Ebool\_2EIN A\_27b) V0y) (ap (ap (c\_2Epred\_set\_2EIMAGE \\
& A\_27a A\_27b) V2f) V1s))) \Leftrightarrow (\exists V3x \in A\_27a. ((V0y = (ap V2f V3x)) \wedge \\
& (p (ap (ap (c\_2Ebool\_2EIN A\_27a) V3x) V1s))))))) \\
\end{aligned} \tag{53}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{54}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{55}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{56}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \tag{57}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q))))) \quad (58)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0f \in (ty\_2Eextreal\_2Eextreal^{ty\_2Enum\_2Enum}). ((\forall V1m \in \\ & ty\_2Enum\_2Enum. (\forall V2n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\ & V1m) V2n)) \Rightarrow (p (ap (ap c\_2Eextreal\_2Eextreal\_le (ap V0f V2n)) ( \\ & ap V0f V1m))))))) \Rightarrow ((ap c\_2Eextreal\_2Eextreal\_inf (ap (ap (c\_2Epred\_set\_2EIMAGE \\ & ty\_2Enum\_2Enum ty\_2Eextreal\_2Eextreal) (\lambda V3n \in ty\_2Enum\_2Enum. \\ & (ap V0f (ap c\_2Enum\_2ESUC V3n))) (c\_2Epred\_set\_2EUNIV ty\_2Enum\_2Enum))) = \\ & (ap c\_2Eextreal\_2Eextreal\_inf (ap (ap (c\_2Epred\_set\_2EIMAGE \\ & ty\_2Enum\_2Enum ty\_2Eextreal\_2Eextreal) V0f) (c\_2Epred\_set\_2EUNIV \\ & ty\_2Enum\_2Enum))))))) \end{aligned}$$