thm_2Eextreal_2Emax__le2__imp (TMVjw9E2ATvjmN6HySBU1VqXwwAuMDJmK5T)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Eextreal\_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eextreal\_2Eextreal \tag{1}$$

Let $c\_2Eextreal\_2Eextreal\_\_le : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_\_le \in ((2^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \tag{2}$$

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 11 *We define* c_2Eextreal_2Eextreal__max *to be* $\lambda V0x \in ty\_2Eextreal\_2Eextreal.\lambda V1y \in ty\_2E$

Assume the following.

$$True \tag{3}$$

Assume the following.

$$(\forall V0t \in 2.(((True \lor (p\ V0t)) \Leftrightarrow True) \land ((((p\ V0t) \lor True) \Leftrightarrow True) \land (((False \lor (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \lor False) \Leftrightarrow (p\ V0t)) \land (((p\ V0t) \lor (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{4}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{5}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \land (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \land (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{6}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \land (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{7}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \land ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{8}$$

Assume the following.

$$(\forall V0z \in ty\_2Eextreal\_2Eextreal.(\forall V1x \in ty\_2Eextreal\_2Eextreal.(\forall V2y \in ty\_2Eextreal\_2Eextreal.((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V0z)\ (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_max}\ V1x)\ V2y))) \Leftrightarrow ((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V0z)\ V1x)) \lor (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V0z)\ V2y))))))) \tag{9}$$

Assume the following.

$$(\forall V0z \in ty\_2Eextreal\_2Eextreal.(\forall V1x \in ty\_2Eextreal\_2Eextreal.(\forall V2y \in ty\_2Eextreal\_2Eextreal.((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_max}\ V1x)\ V2y))\ V0z)) \Leftrightarrow ((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V1x)\ V0z)) \land (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V2y)\ V0z))))))) \tag{10}$$

Theorem 1

$$(\forall V0x1 \in ty\_2Eextreal\_2Eextreal.(\forall V1x2 \in ty\_2Eextreal\_2Eextreal.(\forall V2y1 \in ty\_2Eextreal\_2Eextreal.(\forall V3y2 \in ty\_2Eextreal\_2Eextreal.(((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V0x1)\ V2y1)) \wedge (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V1x2)\ V3y2))) \Rightarrow (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_max}\ V0x1)\ V1x2))\ (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_max}\ V2y1)\ V3y2))))))))$$