thm_2Eextreal_2Emin_infty (TMRyntvDpSJZj9Y1rDSvPhsJC67AkFUCkGb)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P{\Rightarrow}p\ Q)$
of type ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota{\Rightarrow}\iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in 2$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ $c_2Emin_2E_3D_3D_3E $V0t)\ $c_2Ebool_2EF

Let $ty\_2Eextreal\_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eextreal\_2Eextreal \tag{1}$$

Let $c\_2Eextreal\_2EPosInf : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2EPosInf \in ty\_2Eextreal\_2Eextreal \tag{2}$$

Let $c\_2Eextreal\_2ENegInf : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2ENegInf \in ty\_2Eextreal\_2Eextreal \tag{3}$$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Let $c\_2Eextreal\_2Eextreal\_\_le : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_\_le \in ((2^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \tag{4}$$

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$
of type $\iota{\Rightarrow}\iota$.

Definition 10 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 11 *We define* c_2Eextreal_2Eextreal__min *to be* $\lambda V0x \in ty\_2Eextreal\_2Eextreal.\lambda V1y \in ty\_2Ee$

Assume the following.
$$True \tag{5}$$

Assume the following.
$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2))))))) \tag{6}$$

Assume the following.
$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{7}$$

Assume the following.
$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \tag{8}$$

Assume the following.
$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{9}$$

Assume the following.
$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{10}$$

Assume the following.
$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t))))))) \tag{11}$$

Assume the following.
$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in$$
$$A\_27a.(((ap\ (ap\ (ap\ \mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)$$
$$V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ \mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})$$
$$V0t1)\ V1t2) = V1t2)))) \tag{12}$$

Assume the following.
$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0b \in 2.(\forall V1t \in A\_27a.$$
$$((ap\ (ap\ (ap\ \mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V0b)\ V1t)\ V1t) = V1t))) \tag{13}$$

Assume the following.
$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{14}$$

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Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.$$
$$(\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a.$$
$$(\forall V5y\_27 \in A\_27a.((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge$$
$$((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)$$
$$V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)$$
$$V5y\_27)))))))))$$

$$(15)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0t1 \in A\_27a.(\forall V1t2 \in$$
$$A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)$$
$$V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a.(\forall V3t2 \in A\_27a.((ap$$
$$(ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V2t1)\ V3t2) = V3t2))))$$

$$(16)$$

Assume the following.

$$((\forall V0x \in ty\_2Eextreal\_2Eextreal.((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le$$
$$c\_2Eextreal\_2ENegInf)\ V0x)) \wedge (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le$$
$$V0x)\ c\_2Eextreal\_2EPosInf)))) \wedge ((\forall V1x \in ty\_2Eextreal\_2Eextreal.$$
$$((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V1x)\ c\_2Eextreal\_2ENegInf)) \Leftrightarrow$$
$$(V1x = c\_2Eextreal\_2ENegInf))) \wedge (\forall V2x \in ty\_2Eextreal\_2Eextreal.$$
$$((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ c\_2Eextreal\_2EPosInf)$$
$$V2x)) \Leftrightarrow (V2x = c\_2Eextreal\_2EPosInf)))))$$

$$(17)$$

Theorem 1

$$(\forall V0x \in ty\_2Eextreal\_2Eextreal.(((ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_min}$$
$$V0x)\ c\_2Eextreal\_2EPosInf) = V0x) \wedge (((ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_min}$$
$$c\_2Eextreal\_2EPosInf)\ V0x) = V0x) \wedge (((ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_min}$$
$$c\_2Eextreal\_2ENegInf)\ V0x) = c\_2Eextreal\_2ENegInf) \wedge ((ap\ (ap$$
$$\mathsf{c\_2Eextreal\_2Eextreal\_\_min}\ V0x)\ c\_2Eextreal\_2ENegInf) = c\_2Eextreal\_2ENegInf)))))$$