

# thm\_2Efcp\_2ECART\_EQ

(TMYH8VQEFNCCCTUmkNMoXP7RQLxyetmg3EJ)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0x \in A\_27a. (\lambda V1y \in A\_27b. V0x))$

**Definition 3** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. (\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 4** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota. (ap (ap (c\_2Ecombin\_2ES A\_27a (A\_27a^{A\_27a}) A\_27b) A\_27c) A\_27b)$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let  $c\_2Epred\_set\_2ECARD : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Epred\_set\_2ECARD\ A\_27a \in (ty\_2Enum\_2Enum^{(2^{A\_27a})}) \quad (2)$$

**Definition 5** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. (\lambda V1f \in (2^{A\_27a}). (ap V1f V0x)))$

**Definition 6** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p \ P \Rightarrow p \ Q)$  of type  $\iota$ .

**Definition 7** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

**Definition 8** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}) A\_27a) A\_27b) A\_27c))$

**Definition 9** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2. inj\_o (V0t1 \ P \Rightarrow V1t2 \ P))))$

**Definition 10** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2. inj\_o (V0t1 \ P \Rightarrow V1t2 \ P))))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (3)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a &\Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a \ A\_27b &\in ((ty\_2Epair\_2Eprod \ A\_27a \ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (4)$$

**Definition 11** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap (c\_2$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a &\Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a \ A\_27b &\in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod \ A\_27a \ 2)^{A\_27b}}) \end{aligned} \quad (5)$$

**Definition 12** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. \lambda V1s \in (2^{A\_27a}). (ap (c\_2$

**Definition 13** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 \ 2)) (\lambda V0t \in 2.V0t)$ .

**Definition 14** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. c\_2Ebool\_2EF)$ .

**Definition 15** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). (ap (c\_2Ebool\_2E\_21 \ 2))$

**Definition 16** We define  $c\_2Efcp\_2EHAS\_SIZE$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow c\_2Ebool\_2EARB \ A\_27a \in A\_27a \quad (6)$$

Let  $ty\_2Efcp\_2Efinite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \text{nonempty } (ty\_2Efcp\_2Efinite\_image \ A0) \quad (7)$$

Let  $c\_2Efcp\_2Edest\_finite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a &\Rightarrow c\_2Efcp\_2Edest\_finite\_image \\ A\_27a &\in (A\_27a^{(ty\_2Efcp\_2Efinite\_image \ A\_27a)}) \end{aligned} \quad (8)$$

Let  $c\_2Efcp\_2Emk\_finite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a &\Rightarrow c\_2Efcp\_2Emk\_finite\_image \ A\_27a \in \\ ((ty\_2Efcp\_2Efinite\_image \ A\_27a))^{A\_27a} & \end{aligned} \quad (9)$$

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \text{nonempty } (ty\_2Ebool\_2Eitself \ A0) \quad (10)$$

Let  $c\_2Ebool\_2Ethethe\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a &\Rightarrow c\_2Ebool\_2Ethethe\_value \ A\_27a \in \\ ty\_2Ebool\_2Eitself \ A\_27a & \end{aligned} \quad (11)$$

Let  $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow c\_2Efcp\_2Edimindex \ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself \ A\_27a)}) \quad (12)$$

**Definition 17** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\lambda x. x \in A \wedge P x) \text{ else } \perp$

**Definition 18** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (ap V0P (ap (c\_2Emin\_2E\_40 A) a)))$

**Definition 19** We define  $c\_2Ebool\_2E\_3F\_21$  to be  $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (ap (ap (c\_2Ebool\_2E\_2F\_5C A) a)))$

Let  $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty\_2Efcp\_2Ecart A0 A1) \quad (13)$$

Let  $c\_2Efcp\_2Emk\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow \forall A\_27b. nonempty A\_27b \Rightarrow c\_2Efcp\_2Emk\_cart A\_27a A\_27b \in ((ty\_2Efcp\_2Ecart A\_27a A\_27b)^{(A\_27a(ty\_2Efcp\_2Efinite\_image A\_27b))}) \quad (14)$$

**Definition 20** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2. (ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (15)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (16)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (17)$$

**Definition 21** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap c\_2Enum\_2EABS\_num m)$

**Definition 22** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. (ap c\_2Enum\_2ESUC (V0m, V1n))$

**Definition 23** We define  $c\_2Efcp\_2Efinite\_index$  to be  $\lambda A\_27a : \iota. (ap (c\_2Emin\_2E\_40 (A\_27a^{ty\_2Enum\_2Enum})))$

Let  $c\_2Efcp\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow \forall A\_27b. nonempty A\_27b \Rightarrow c\_2Efcp\_2Edest\_cart A\_27a A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_image A\_27b)})^{(ty\_2Efcp\_2Ecart A\_27a A\_27b)}) \quad (18)$$

**Definition 24** We define  $c\_2Efcp\_2Efcp\_index$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in (ty\_2Efcp\_2Ecart A\_27a A\_27b). (ap (c\_2Efcp\_2Edest\_cart A\_27a A\_27b) x)$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (19)$$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (20)$$

**Definition 25** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 26** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (21)$$

**Definition 27** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP).$

**Definition 28** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

**Definition 29** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ (ap\ (ap\ (ap\ (c\_2Ebool\_2B$

Let  $c\_2Earithmetic\_2EEEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (22)$$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (23)$$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (24)$$

**Definition 30** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x.$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (25)$$

**Definition 31** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_2Earithmetic\_2E$

**Definition 32** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 33** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. c\_2Ebool\_2ET).$

**Definition 34** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap\ (c\_2E$

**Definition 35** We define  $c\_2Epred\_set\_2EDELETE$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1x \in A\_27a. (ap\ (a$

**Definition 36** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A\_27a}). \lambda V1s \in$

**Definition 37** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0.$

**Definition 38** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_2Earithmetic\_2E$

**Definition 39** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x.$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & (ap c\_2Enum\_2ESUC V0m)) V1n)))) \\
 \end{aligned} \tag{26}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 c\_2Enum\_2E0) V0n))) \tag{27}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge \\
 & (((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\
 & (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL \\
 & (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge \\
 & (((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL \\
 & (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge \\
 & ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap \\
 & (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) \\
 & V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = \\
 & (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A \\
 & V0m) V1n))))))) \\
 \end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & \forall V2p \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))) \Rightarrow (p ( \\
 & ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V2p)))))) \\
 \end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & (V0m = V1n) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \wedge (p ( \\
 & ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V0m)))))) \\
 \end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) (ap (ap c\_2Earithmetic\_2E\_2B \\
 & V0m) V2p))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p)))))) \\
 \end{aligned} \tag{31}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ c\_2Enum\_2ESUC\ V0n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))\ V0n))) \quad (32)$$

Assume the following.

$$True \quad (33)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (36)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (37)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (38)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge (p\ V1t2) \wedge (p\ V2t3)) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2) \wedge (p\ V2t3))) \wedge (p\ V2t3)))))) \quad (39)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \quad (40)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (41)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (42)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t))))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow true) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \end{aligned} \quad (44)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True))) \quad (45))$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\ True)) \quad (46)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\ A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (47)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow \\ & \forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27b^{A\_27a}).((V0f = \\ & V1g) \Leftrightarrow (\forall V2x \in A\_27a.((ap V0f V2x) = (ap V1g V2x)))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ & (p V0t))))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in \\ & A\_27a.(((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) \\ & V1t2) = V0t1) \wedge ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) \\ & V0t1) V1t2) = V1t2)))))) \end{aligned} \quad (50)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\forall V1x \in \\ A\_27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A\_27a.(\neg(p (ap V0P V2x))))))) \quad (51)$$

Assume the following.

$$\forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0P \in (2^{A_{\text{27a}}})). ((\neg(\exists V1x \in A_{\text{27a}}.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A_{\text{27a}}.(\neg(p (ap V0P V2x))))) \quad (52)$$

Assume the following.

$$\begin{aligned} \forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow & (\forall V0P \in (2^{A_{\text{27a}}})). (\forall V1Q \in (2^{A_{\text{27a}}})). ((\forall V2x \in A_{\text{27a}}.((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow \\ & ((\forall V3x \in A_{\text{27a}}.((p (ap V0P V3x)) \wedge (\forall V4x \in A_{\text{27a}}.((p (ap V1Q V4x))))))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} \forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow & (\forall V0P \in 2). (\forall V1Q \in (2^{A_{\text{27a}}})). (((p V0P) \wedge (\forall V2x \in A_{\text{27a}}.((p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A_{\text{27a}}.((p V0P) \wedge (p (ap V1Q V3x))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} \forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow & (\forall V0P \in (2^{A_{\text{27a}}})). (\forall V1Q \in 2). (((\exists V2x \in A_{\text{27a}}.((p (ap V0P V2x)) \vee (p V1Q))) \Leftrightarrow (\exists V3x \in A_{\text{27a}}.((p (ap V0P V3x)) \vee (p V1Q)))))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} \forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow & (\forall V0P \in 2). (\forall V1Q \in (2^{A_{\text{27a}}})). (((p V0P) \vee (\exists V2x \in A_{\text{27a}}.((p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in A_{\text{27a}}.((p V0P) \vee (p (ap V1Q V3x))))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned} \forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow & (\forall V0P \in (2^{A_{\text{27a}}})). (\forall V1Q \in (2^{A_{\text{27a}}})). (((\exists V2x \in A_{\text{27a}}.((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in A_{\text{27a}}.((p (ap V0P V3x)) \wedge (p V1Q)))))) \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} \forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow & (\forall V0P \in 2). (\forall V1Q \in (2^{A_{\text{27a}}})). (((\exists V2x \in A_{\text{27a}}.((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \wedge (\exists V3x \in A_{\text{27a}}.((p (ap V1Q V3x))))))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} \forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow & (\forall V0P \in 2). (\forall V1Q \in (2^{A_{\text{27a}}})). (((\forall V2x \in A_{\text{27a}}.((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A_{\text{27a}}.((p (ap V1Q V3x))))))) \end{aligned} \quad (59)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. ((\neg((p V0A) \Rightarrow (p V1B))) \Leftrightarrow ((p V0A) \wedge (\neg(p V1B))))) \quad (60)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B)) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))) \quad (61)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (62)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (63)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(p V0A) \wedge (p V1B)) \Leftrightarrow ((\neg(p V0A) \vee (p V1B)) \Leftrightarrow ((\neg(p V0A) \wedge (\neg(p V1B))))))) \quad (64)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))) \quad (65)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (66)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (67)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow \\ & \forall V0b \in 2. (\forall V1f \in (A_{27b}^{A_{27a}}). (\forall V2g \in (A_{27b}^{A_{27a}}). \\ & (\forall V3x \in A_{27a}. ((ap (ap (ap (ap (c_2Ebool\_2ECOND (A_{27b}^{A_{27a}}) \\ & V0b) V1f) V2g) V3x) = (ap (ap (ap (c_2Ebool\_2ECOND A_{27b}) V0b) (ap \\ & V1f V3x)) (ap V2g V3x))))))) \end{aligned} \quad (68)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow \\ & \forall V0f \in (A_{27b}^{A_{27a}}). (\forall V1b \in 2. (\forall V2x \in A_{27a}. \\ & (\forall V3y \in A_{27a}. ((ap V0f (ap (ap (ap (c_2Ebool\_2ECOND A_{27a}) \\ & V1b) V2x) V3y)) = (ap (ap (ap (c_2Ebool\_2ECOND A_{27b}) V1b) (ap V0f \\ & V2x)) (ap V0f V3y))))))) \end{aligned} \quad (69)$$

Assume the following.

$$\begin{aligned} & (\forall V0b \in 2. (\forall V1t1 \in 2. (\forall V2t2 \in 2. ((p (ap (ap \\ & (ap (c_2Ebool_2ECOND 2) V0b) V1t1) V2t2)) \Leftrightarrow (((\neg(p V0b)) \vee (p V1t1)) \wedge \\ & ((p V0b) \vee (p V2t2))))))) \end{aligned} \quad (70)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. nonempty A_{27a} \Rightarrow (\forall V0P \in (2^{A_{27a}}). ((p (ap \\ & (c_2Ebool_2E_3F_21 A_{27a}) (\lambda V1x \in A_{27a}. (ap V0P V1x)))) \Leftrightarrow (( \\ & \exists V2x \in A_{27a}. (p (ap V0P V2x))) \wedge (\forall V3x \in A_{27a}. (\forall V4y \in \\ & A_{27a}. (((p (ap V0P V3x)) \wedge (p (ap V0P V4y))) \Rightarrow (V3x = V4y))))))) \end{aligned} \quad (71)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_{27} \in 2. (\forall V2y \in 2. (\forall V3y_{27} \in \\ & 2. (((((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27}))))))) \end{aligned} \quad (72)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. nonempty A_{27a} \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A_{27a}. (\forall V3x_{27} \in A_{27a}. (\forall V4y \in A_{27a}. \\ & (\forall V5y_{27} \in A_{27a}. (((((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_{27})) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_{27})))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND A_{27a} \\ & V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND A_{27a}) V1Q) V3x_{27}) \\ & V5y_{27}))))))))))) \end{aligned} \quad (73)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. nonempty A_{27a} \Rightarrow (\forall V0f \in (2^{A_{27a}}). (\forall V1v \in \\ & A_{27a}. ((\forall V2x \in A_{27a}. ((V2x = V1v) \Rightarrow (p (ap V0f V2x)))) \Leftrightarrow (p ( \\ & ap V0f V1v)))))) \end{aligned} \quad (74)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. nonempty A_{27a} \Rightarrow \forall A_{27b}. nonempty A_{27b} \Rightarrow ( \\ & \forall V0P \in ((2^{A_{27b}})^{A_{27a}}). ((\forall V1x \in A_{27a}. (\exists V2y \in \\ & A_{27b}. (p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A_{27b})^{A_{27a}}). ( \\ & \forall V4x \in A_{27a}. (p (ap (ap V0P V4x) (ap V3f V4x))))))) \end{aligned} \quad (75)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. nonempty A_{27a} \Rightarrow (\forall V0x \in A_{27a}. ((ap (c_2Ecombin_2EI \\ & A_{27a}) V0x) = V0x)) \end{aligned} \quad (76)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & ((\forall V0a \in (ty\_2Efcp\_2Efinit\_image \\ A_{27a}).((ap\ (c\_2Efcp\_2Emk\_finite\_image\ A_{27a})\ (ap\ (c\_2Efcp\_2Edest\_finite\_image\\ A_{27a})\ V0a)) = V0a)) \wedge (\forall V1r \in A_{27a}.((p\ (ap\ (\lambda V2x \in A_{27a}.\\ (ap\ (ap\ c\_2Ebool\_2E\_5C\_2F\ (ap\ (ap\ (c\_2Emin\_2E\_3D\ A_{27a})\ V2x)\ (c\_2Ebool\_2EARB\\ A_{27a}))\ (ap\ (c\_2Epred\_set\_2EFINITE\ A_{27a})\ (c\_2Epred\_set\_2EUNIV\\ A_{27a})))\ V1r)) \Leftrightarrow ((ap\ (c\_2Efcp\_2Edest\_finite\_image\ A_{27a})\\ (ap\ (c\_2Efcp\_2Emk\_finite\_image\ A_{27a})\ V1r)) = V1r)))))) \\ (77) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & ((ap\ (c\_2Efcp\_2Edimindex\ A_{27a})\\ (c\_2Ebool\_2Ethe\_value\ A_{27a})) = (ap\ (ap\ (c\_2Ebool\_2ECOND\\ ty\_2Enum\_2Enum)\ (ap\ (c\_2Epred\_set\_2EFINITE\ A_{27a})\ (c\_2Epred\_set\_2EUNIV\\ A_{27a})))\ (ap\ (c\_2Epred\_set\_2ECARD\ A_{27a})\ (c\_2Epred\_set\_2EUNIV\\ A_{27a})))\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\\ c\_2Earithmetic\_2EZERO)))))) \\ (78) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & \forall A_{27b}.nonempty\ A_{27b} \Rightarrow \\ & ((\forall V0a \in (ty\_2Efcp\_2Ecart\ A_{27a}\ A_{27b}).((ap\ (c\_2Efcp\_2Emk\_cart\\ A_{27a}\ A_{27b})\ (ap\ (c\_2Efcp\_2Edest\_cart\ A_{27a}\ A_{27b})\ V0a)) = V0a)) \wedge \\ & (\forall V1r \in (A_{27a}^{(ty\_2Efcp\_2Efinit\_image\ A_{27b})}).((p\ (\\ ap\ (\lambda V2f \in (A_{27a}^{(ty\_2Efcp\_2Efinit\_image\ A_{27b})}).c\_2Ebool\_2ET)\\ V1r)) \Leftrightarrow ((ap\ (c\_2Efcp\_2Edest\_cart\ A_{27a}\ A_{27b})\ (ap\ (c\_2Efcp\_2Emk\_cart\\ A_{27a}\ A_{27b})\ V1r)) = V1r)))))) \\ (79) \end{aligned}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg((ap\ c\_2Enum\_2ESUC\ V0n) = c\_2Enum\_2E0))) \\ (80)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ c\_2Enum\_2E0)) \wedge \\ & (\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c\_2Enum\_2ESUC\\ V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n)))))) \\ (81) \end{aligned}$$

Assume the following.

$((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap (ap c\_2Earithmetic\_2E\_2B V2n) V3m))))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A V6n) V7m))))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D V10n) V11m))))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEEXP V14n) c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))))) \wedge ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2EEEXP V15n) V16m))))))) \wedge (((ap c\_2Enum\_2ESUC c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum.((ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enum\_2ESUC V17n))))))) \wedge (((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Eprim\_rec\_2EPRE V18n))))))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum.((\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m))))))) \wedge ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C V23n) c\_2Enum\_2E0)) \Leftrightarrow False))) \wedge ((\forall V24n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) V24n))))))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.((\forall V26m \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C V25n) c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL V26m)))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C V25n) V26m))))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E c\_2Enum\_2E0) V27n)) \Leftrightarrow False))) \wedge ((\forall V28n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E V28n) c\_2Enum\_2E0) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3E c\_2Enum\_2E0) V28n))))))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.((\forall V30m \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C V29n) c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL V30m)))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C V29n) V30m))))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C c\_2Enum\_2E0) V31n)) \Leftrightarrow True))) \wedge ((\forall V32n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C c\_2Enum\_2E0) V32n)) \Leftrightarrow False))) \wedge ((p (ap (ap c\_2Earithmetic\_2E\_3C c\_2Enum\_2E0) V32n)) \Leftrightarrow True)))$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D c_2Earithmetic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2EBIT2 V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n))) \wedge ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))))))))))) \\
\end{aligned} \tag{83}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\
& (2^{A\_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a. ((p (ap (ap (c_2Ebool\_2EIN \\
& A\_27a) V2x) V0s)) \Leftrightarrow (p (ap (ap (c_2Ebool\_2EIN A\_27a) V2x) V1t))))))) \\
\end{aligned} \tag{84}$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. (\neg(p (ap (ap \\
(c_2Ebool\_2EIN A\_27a) V0x) (c_2Epred\_set\_2EEMPTY A\_27a))))) \tag{85}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). ((\exists V1x \in \\
& A\_27a. (p (ap (ap (c_2Ebool\_2EIN A\_27a) V1x) V0s)) \Leftrightarrow (\neg(V0s = (c_2Epred\_set\_2EEMPTY \\
& A\_27a))))) \\
\end{aligned} \tag{86}$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. (p (ap (ap (c_2Ebool\_2EIN \\
A\_27a) V0x) (c_2Epred\_set\_2EUNIV A\_27a)))) \tag{87}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in \\
& A\_27a. (\forall V2s \in (2^{A\_27a}). ((p (ap (ap (c_2Ebool\_2EIN A\_27a) \\
& V0x) (ap (ap (c_2Epred\_set\_2EINSERT A\_27a) V1y) V2s))) \Leftrightarrow ((V0x = \\
& V1y) \vee (p (ap (ap (c_2Ebool\_2EIN A\_27a) V0x) V2s))))))) \\
\end{aligned} \tag{88}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1s \in \\
& (2^{A\_27a}). (\neg((p (ap (ap (c_2Epred\_set\_2EINSERT A\_27a) V0x) V1s) \\
& (c_2Epred\_set\_2EEMPTY A\_27a)))))) \\
\end{aligned} \tag{89}$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty A_{27a} \Rightarrow & (\forall V0s \in (2^{A_{27a}}).(\forall V1x \in \\ A_{27a}.(\forall V2y \in A_{27a}.((p(ap(ap(c_2Ebool_2EIN A_{27a}) V1x) \\ (ap(ap(c_2Epred_set_2EDELETE A_{27a}) V0s) V2y))) \Leftrightarrow ((p(ap(ap(c_2Ebool_2EIN A_{27a}) V1x) V0s)) \wedge (\neg(V1x = V2y))))))) \end{aligned} \quad (90)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty A_{27a} \Rightarrow & (\forall V0x \in A_{27a}.(\forall V1s \in \\ (2^{A_{27a}}).((p(ap(ap(c_2Ebool_2EIN A_{27a}) V0x) V1s)) \Rightarrow ((ap(ap(c_2Epred_set_2EINSERT A_{27a}) V0x) (ap(ap(c_2Epred_set_2EDELETE \\ A_{27a}) V1s) V0x)) = V1s)))))) \end{aligned} \quad (91)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty A_{27a} \Rightarrow & (\forall V0x \in A_{27a}.(\forall V1y \in \\ A_{27a}.((p(ap(ap(c_2Ebool_2EIN A_{27a}) V0x) (ap(ap(c_2Epred_set_2EINSERT \\ A_{27a}) V1y) (c_2Epred_set_2EEMPTY A_{27a}))) \Leftrightarrow (V0x = V1y)))))) \end{aligned} \quad (92)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty A_{27a} \Rightarrow & \forall A_{27b}.nonempty A_{27b} \Rightarrow \\ & (\forall V0y \in A_{27b}.(\forall V1s \in (2^{A_{27a}}).(\forall V2f \in (A_{27b}^{A_{27a}}). \\ ((p(ap(ap(c_2Ebool_2EIN A_{27b}) V0y) (ap(ap(c_2Epred_set_2EIMAGE \\ A_{27a} A_{27b}) V2f) V1s))) \Leftrightarrow (\exists V3x \in A_{27a}.((V0y = (ap V2f V3x)) \wedge \\ (p(ap(ap(c_2Ebool_2EIN A_{27a}) V3x) V1s))))))) \end{aligned} \quad (93)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty A_{27a} \Rightarrow & \forall A_{27b}.nonempty A_{27b} \Rightarrow \\ & (\forall V0f \in (A_{27b}^{A_{27a}}).((ap(ap(c_2Epred_set_2EIMAGE A_{27a} \\ A_{27b}) V0f) (c_2Epred_set_2EEMPTY A_{27a})) = (c_2Epred_set_2EEMPTY \\ A_{27b}))) \end{aligned} \quad (94)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty A_{27a} \Rightarrow & \forall A_{27b}.nonempty A_{27b} \Rightarrow \\ & (\forall V0f \in (A_{27b}^{A_{27a}}).(\forall V1x \in A_{27a}.(\forall V2s \in ( \\ 2^{A_{27a}}).((ap(ap(c_2Epred_set_2EIMAGE A_{27a} A_{27b}) V0f) (ap \\ (ap(c_2Epred_set_2EINSERT A_{27a}) V1x) V2s)) = (ap(ap(c_2Epred_set_2EINSERT \\ A_{27b}) (ap V0f V1x)) (ap(ap(c_2Epred_set_2EIMAGE A_{27a} A_{27b}) \\ V0f) V2s))))))) \end{aligned} \quad (95)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (p(ap(c_2Epred_set_2EFINITE \\ A_{27a}) (c_2Epred_set_2EEMPTY A_{27a}))) \quad (96)$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0P \in (2^{(2^{A_{27a}})}).(( \\
& (p (ap V0P (c_2Epred_set_2EEMPTY A_{27a}))) \wedge (\forall V1s \in (2^{A_{27a}}). \\
& (((p (ap (c_2Epred_set_2EFINITE A_{27a}) V1s)) \wedge (p (ap V0P V1s))) \Rightarrow \\
& (\forall V2e \in A_{27a}.((\neg(p (ap (c_2Ebool_2EIN A_{27a}) V2e) V1s))) \Rightarrow \\
& (p (ap V0P (ap (ap (c_2Epred_set_2EINSERT A_{27a}) V2e) V1s))))))) \Rightarrow \\
& (\forall V3s \in (2^{A_{27a}}).((p (ap (c_2Epred_set_2EFINITE A_{27a}) \\
& V3s)) \Rightarrow (p (ap V0P V3s)))))) \\
\end{aligned} \tag{97}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0x \in A_{27a}.(\forall V1s \in \\
& (2^{A_{27a}}).((p (ap (c_2Epred_set_2EFINITE A_{27a}) (ap (ap (c_2Epred_set_2EINSERT \\
& A_{27a}) V0x) V1s))) \Leftrightarrow (p (ap (c_2Epred_set_2EFINITE A_{27a}) V1s)))))) \\
\end{aligned} \tag{98}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0x \in A_{27a}.(\forall V1s \in \\
& (2^{A_{27a}}).((p (ap (c_2Epred_set_2EFINITE A_{27a}) (ap (ap (c_2Epred_set_2EDELETE \\
& A_{27a}) V1s) V0x))) \Leftrightarrow (p (ap (c_2Epred_set_2EFINITE A_{27a}) V1s)))))) \\
\end{aligned} \tag{99}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0x \in A_{27a}.(p (ap (c_2Epred_set_2EFINITE \\
& A_{27a}) (ap (ap (c_2Epred_set_2EINSERT A_{27a}) V0x) (c_2Epred_set_2EEMPTY \\
& A_{27a})))))) \\
\end{aligned} \tag{100}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow \\
& \forall V0s \in (2^{A_{27a}}).((p (ap (c_2Epred_set_2EFINITE A_{27a}) \\
& V0s)) \Rightarrow (\forall V1f \in (A_{27b}^{A_{27a}}).(p (ap (c_2Epred_set_2EFINITE \\
& A_{27b}) (ap (ap (c_2Epred_set_2EIMAGE A_{27a} A_{27b}) V1f) V0s)))))) \\
\end{aligned} \tag{101}$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow ((ap (c_2Epred_set_2ECARD A_{27a}) \\
(c_2Epred_set_2EEMPTY A_{27a})) = c_2Enum_2E0) \tag{102}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0s \in (2^{A_{27a}}).((p (ap \\
& (c_2Epred_set_2EFINITE A_{27a}) V0s)) \Rightarrow (\forall V1x \in A_{27a}.(( \\
& ap (c_2Epred_set_2ECARD A_{27a}) (ap (ap (c_2Epred_set_2EINSERT \\
& A_{27a}) V1x) V0s)) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) \\
& (ap (ap (c_2Ebool_2EIN A_{27a}) V1x) V0s)) (ap (c_2Epred_set_2ECARD \\
& A_{27a}) V0s)) (ap c_2Enum_2ESUC (ap (c_2Epred_set_2ECARD A_{27a}) \\
& V0s))))))) \\
\end{aligned} \tag{103}$$

Assume the following.

$$\begin{aligned} \forall A.27a.\text{nonempty } A.27a \Rightarrow (\forall V0x \in A.27a.((ap(c.2Epred\_set\_2ECARD \\ A.27a) (ap(ap(c.2Epred\_set\_2EINSERT A.27a) V0x) (c.2Epred\_set\_2EEMPTY \\ A.27a))) = (ap(c.2Earithmetic\_2ENUMERAL (ap(c.2Earithmetic\_2EBIT1 \\ c.2Earithmetic\_2EZERO)))))) \end{aligned} \quad (104)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum. \\ ((ap(c.2Enum\_2ESUC V0m) = (ap(c.2Enum\_2ESUC V1n)) \Leftrightarrow (V0m = V1n)))) \quad (105)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg(p(ap(ap(c.2Eprim\_rec\_2E\_3C \\ V0n) V0n)))))) \quad (106)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum. \\ (p(ap(ap(c.2Eprim\_rec\_2E\_3C V0m) (ap(c.2Enum\_2ESUC V1n))) \Leftrightarrow \\ (V0m = V1n) \vee (p(ap(ap(c.2Eprim\_rec\_2E\_3C V0m) V1n))))))) \quad (107)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (108)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (109)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\ (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (110)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\ (((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (111)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (112)$$

Assume the following.

$$\begin{aligned} &(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ &(p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ &((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (113)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))) \end{aligned} \quad (114)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (115)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (( \\ & (\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (116)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (117)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (118)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (119)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (120)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (121)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (122)$$

### Theorem 1

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow \\ & \forall V0x \in (\text{ty\_2Efcp\_2Ecart } A\_27a \_A\_27b). (\forall V1y \in (\text{ty\_2Efcp\_2Ecart } \\ & A\_27a \_A\_27b). ((V0x = V1y) \Leftrightarrow (\forall V2i \in \text{ty\_2Enum\_2Enum}. ((p \_ap \\ & (\_ap \_c\_2Eprim\_rec\_2E\_3C \_V2i) \_ap (\_c\_2Efcp\_2Edimindex \_A\_27b) \_ \\ & c\_2Ebool\_2Ethe\_value \_A\_27b))) \Rightarrow ((\_ap \_ap (\_c\_2Efcp\_2Efcpc\_index \\ & A\_27a \_A\_27b) \_V0x) \_V2i) = (\_ap \_ap (\_c\_2Efcp\_2Efcpc\_index \_A\_27a \_A\_27b) \\ & \_V1y) \_V2i))) )) \end{aligned}$$