

thm_2Efcp_2EFCP_HD

(TMTz3kqTSfZMC48Ftih2byYjVqH5dGB57Ns)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x))$

Definition 3 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 4 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota. (ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a}) A_27b) A_27c) A_27b)$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Let $c_2Epred_set_2ECARD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Epred_set_2ECARD\ A_27a \in (ty_2Enum_2Enum^{(2^{A_27a})}) \quad (2)$$

Definition 5 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 7 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 8 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a}) A_27a) A_27b) A_27c))$

Definition 9 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. inj_o (V0t1\ t2))))$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. inj_o (V0t1\ t2))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (3)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a \ A_27b \in ((ty_2Epair_2Eprod \ A_27a \ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (4)$$

Definition 11 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a \ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod \ A_27a \ 2)^{A_27b}}) \end{aligned} \quad (5)$$

Definition 12 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap (c_2$

Definition 13 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 \ 2) (\lambda V0t \in 2.V0t))$.

Definition 14 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Definition 15 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap (c_2Ebool_2E_21 (2$

Definition 16 We define $c_2Efcp_2EHAS_SIZE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1n \in ty_2Enum_2Env$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Ebool_2EARB \ A_27a \in A_27a \quad (6)$$

Let $ty_2Efcp_2Efinit_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow nonempty (ty_2Efcp_2Efinit_image A0) \quad (7)$$

Let $c_2Efcp_2Edest_finite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Efcp_2Edest_finite_image \\ A_27a \in (A_27a^{(ty_2Efcp_2Efinit_image A_27a)}) \end{aligned} \quad (8)$$

Let $c_2Efcp_2Emk_finite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Efcp_2Emk_finite_image \\ A_27a \in ((ty_2Efcp_2Efinit_image A_27a)^{A_27a}) \end{aligned} \quad (9)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (11)$$

Definition 17 We define c_2Enum_2E0 to be $(ap \ c_2Enum_2EABS_num \ c_2Enum_2EZERO_REP)$.

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (12)$$

Let $c_2Ebool_2Ethethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2Ethethe_value A_27a \in (\\ ty_2Ebool_2Eitself A_27a) \end{aligned} \quad (13)$$

Let $c_2Efcp_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Efcp_2Edimindex A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (14)$$

Definition 18 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2Eitself))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (15)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (16)$$

Definition 19 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num m)$

Definition 20 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (\text{the } (\lambda x.x \in A \wedge p(x))) \text{ of type } \iota \Rightarrow \iota$.

Definition 21 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40)))$

Definition 22 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap (c_2Eprim_rec_2E_3C m n))$

Definition 23 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_2Ebool_2E_2F_5C)))$

Definition 24 We define $c_2Efcp_2Efinit_index$ to be $\lambda A_27a : \iota.(ap (c_2Emin_2E_40 (A_27a^{ty_2Enum_2Enum})))$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efcp_2Ecart \\ A0 A1) \end{aligned} \quad (17)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efcp_2Edest_cart \\ A_27a A_27b \in ((A_27a^{(ty_2Efcp_2Efinit_image A_27b)})(ty_2Efcp_2Ecart A_27a A_27b)) \end{aligned} \quad (18)$$

Definition 25 We define $c_2Efcp_2Efcp_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcp_2Ecart A_27a A_27b).(\omega^{(ty_2Efcp_2Efinit_image A_27b)})$

Definition 26 We define $c_2Efcp_2EFCP_HD$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0v \in (ty_2Efcp_2Ecart A_27a A_27b).(\omega^{(ty_2Efcp_2Efinit_image A_27b)})$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_0.nonempty\ A_0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A_0) \quad (19)$$

Let $c_2Elist_2EGENLIST : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EGENLIST\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{ty_2Enum_2Enum})^{(A_27a^{ty_2Enum_2Enum})}) \quad (20)$$

Definition 27 We define c_2Efcp_2EV2L to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0v \in (ty_2Efcp_2Ecart\ A_27a\ A_27b)$

Let $c_2Elist_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EHD\ A_27a \in (A_27a^{(ty_2Elist_2Elist\ A_27a)}) \quad (21)$$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (22)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (23)$$

Definition 28 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (V0m, V1n)$

Definition 29 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (V0m, V1n)$

Definition 30 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (V0t, V1t1, V2t2))))$

Definition 31 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (V0m, V0m), V0m), V0m), V0m), V0m), V0m))$

Let $c_2Earithmetic_2EEEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (24)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (25)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (26)$$

Definition 32 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (27)$$

Definition 33 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EBIT1) V0n))$

Definition 34 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EBIT2) V0m))$

Definition 35 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 36 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 37 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 38 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EBIT1) V0n))$

Definition 39 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.((\neg(V0n = c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EBIT1) V0n)))) \quad (28)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p (ap (ap c_2Earithmetic_2E_3C_3D c_2Enum_2E0) V0n))) \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum. \\ & ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge \\ & ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\ & (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\ & V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\ & (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\ & V0m) V1n))))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum. \\ & (\forall V2p \in ty_2Enum_2Enum.(((p (ap (ap c_2Earithmetic_2E_3C_3D \\ & V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (\\ & ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p))))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum. \\ & (V0m = V1n) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (\\ & ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m))))))) \end{aligned} \quad (32)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B V0m) V2p)))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))))))) \quad (33)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.((ap\ c_2Enum_2ESUC\ V0n) = (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))\ V0n))) \quad (34)$$

Assume the following.

True (35)

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (37)$$

Assume the following.

$$(\forall V \exists t \in 2. ((p \ V 0t) \vee (\neg(p \ V 0t)))) \quad (38)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (39)$$

Assume the following.

$$((\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.((p V0t1) \wedge (p V1t2) \wedge (p V2t3)))) \Leftrightarrow ((p V0t1) \wedge (p V1t2) \wedge (p V2t3)))))) \quad (40)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \quad (41)$$

Assume the following.

$$(\forall V \exists t \in 2. ((\neg(p \vee 0t)) \Rightarrow ((p \vee 0t) \Rightarrow False))) \quad (42)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \: V0t)) \Leftrightarrow (p \: V0t)) \wedge (((p \: V0t) \wedge True) \Leftrightarrow (p \: V0t)) \wedge (((False \wedge (p \: V0t)) \Leftrightarrow False) \wedge (((p \: V0t) \wedge False) \Leftrightarrow False) \wedge (((p \: V0t) \wedge (p \: V0t)) \Leftrightarrow (p \: V0t))))))) \quad (43)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \end{aligned} \quad (45)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True))) \quad (46))$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0x \in A_{27a}.((V0x = V0x) \Leftrightarrow \\ True)) \quad (47)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0x \in A_{27a}.(\forall V1y \in \\ A_{27a}.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (48)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ & (p V0t))))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0t1 \in A_{27a}.(\forall V1t2 \in \\ & A_{27a}.(((ap (ap (ap (c_2Ebool_2ECOND A_{27a}) c_2Ebool_2ET) V0t1) \\ & V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_{27a}) c_2Ebool_2EF) \\ & V0t1) V1t2) = V1t2)))))) \end{aligned} \quad (50)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0P \in (2^{A_{27a}}).((\neg(\forall V1x \in \\ A_{27a}.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A_{27a}.(\neg(p (ap V0P V2x))))))) \quad (51)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0P \in (2^{A_{27a}}).((\neg(\exists V1x \in \\ A_{27a}.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A_{27a}.(\neg(p (ap V0P V2x))))))) \quad (52)$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow & (\forall V0P \in (2^{A_27a}).(\forall V1Q \in \\ (2^{A_27a}).((\forall V2x \in A_27a.((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow \\ ((\forall V3x \in A_27a.(p (ap V0P V3x))) \wedge (\forall V4x \in A_27a.(p (ap V1Q V4x))))))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow & (\forall V0P \in (2^{A_27a}).(\forall V1Q \in \\ 2.(((\exists V2x \in A_27a.(p (ap V0P V2x))) \vee (p V1Q)) \Leftrightarrow (\exists V3x \in \\ A_27a.((p (ap V0P V3x)) \vee (p V1Q))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow & (\forall V0P \in 2.(\forall V1Q \in (\\ 2^{A_27a}).((\exists V2x \in A_27a.((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p \\ V0P) \wedge (\exists V3x \in A_27a.(p (ap V1Q V3x))))))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow & (\forall V0P \in 2.(\forall V1Q \in (\\ 2^{A_27a}).((\forall V2x \in A_27a.((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p \\ V0P) \vee (\forall V3x \in A_27a.(p (ap V1Q V3x))))))) \end{aligned} \quad (56)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (\\ (p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (57)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (\\ (p V0A)))))) \quad (58)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (\\ (p V0A)))))) \quad (59)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(\\ p V0A)) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p \\ V1B))))))) \end{aligned} \quad (60)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (\\ (p V1B)))))) \quad (61)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (62)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (63)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow \\ & \quad (\forall V0f \in (A_{.27b}^{A_{.27a}}).(\forall V1b \in 2.(\forall V2x \in A_{.27a}. \\ & \quad (\forall V3y \in A_{.27a}.((ap V0f (ap (ap (ap (c_{.2Ebool_2ECOND} A_{.27a}) \\ & \quad V1b) V2x) V3y)) = (ap (ap (ap (c_{.2Ebool_2ECOND} A_{.27b}) V1b) (ap V0f \\ & \quad V2x)) (ap V0f V3y)))))))))) \end{aligned} \quad (64)$$

Assume the following.

$$(\forall V0b \in 2.(\forall V1t1 \in 2.(\forall V2t2 \in 2.((p (ap (ap \\ & \quad (ap (c_{.2Ebool_2ECOND} 2) V0b) V1t1) V2t2)) \Leftrightarrow (((\neg(p V0b)) \vee (p V1t1)) \wedge \\ & \quad ((p V0b) \vee (p V2t2))))))) \quad (65)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in \\ 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow \quad (66) \\ (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))))$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & \quad (\forall V2x \in A_{.27a}.(\forall V3x_{.27} \in A_{.27a}.(\forall V4y \in A_{.27a}. \\ & \quad (\forall V5y_{.27} \in A_{.27a}.(((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_{.27})) \wedge \\ & \quad ((\neg(p V1Q)) \Rightarrow (V4y = V5y_{.27})))))) \Rightarrow ((ap (ap (ap (c_{.2Ebool_2ECOND} A_{.27a}) \\ & \quad V0P) V2x) V4y) = (ap (ap (ap (c_{.2Ebool_2ECOND} A_{.27a}) V1Q) V3x_{.27} \\ & \quad V5y_{.27})))))))))) \end{aligned} \quad (67)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0f \in (2^{A_{.27a}}).(\forall V1v \in \\ & \quad A_{.27a}.((\forall V2x \in A_{.27a}.((V2x = V1v) \Rightarrow (p (ap V0f V2x)))) \Leftrightarrow (p (\\ & \quad ap V0f V1v)))))) \end{aligned} \quad (68)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow \\ & \quad (\forall V0P \in ((2^{A_{.27b}})^{A_{.27a}}).((\forall V1x \in A_{.27a}.(\exists V2y \in \\ & \quad A_{.27b}.(p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A_{.27b}^{A_{.27a}}).(\ \\ & \quad \forall V4x \in A_{.27a}.(p (ap (ap V0P V4x) (ap V3f V4x)))))))))) \end{aligned} \quad (69)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0x \in A_{27a}. ((ap (c_2Ecombin_2EI A_{27a}) V0x) = V0x)) \quad (70)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow & ((\forall V0a \in (ty_2Efcp_2Efinit_image A_{27a}). ((ap (c_2Efcp_2Emk_finite_image A_{27a}) (ap (c_2Efcp_2Edest_finite_image A_{27a}) V0a)) = V0a)) \wedge (\forall V1r \in A_{27a}. ((p (ap (\lambda V2x \in A_{27a}. \\ & (ap (ap c_2Ebool_2E5C_2F (ap (ap (c_2Emin_2E3D A_{27a}) V2x) (c_2Ebool_2EARB A_{27a}))) (ap (c_2Epred_set_2EFINITE A_{27a}) (c_2Epred_set_2EUNIV A_{27a})))) V1r) \Leftrightarrow ((ap (c_2Efcp_2Edest_finite_image A_{27a}) \\ & (ap (c_2Efcp_2Emk_finite_image A_{27a}) V1r)) = V1r)))))) \end{aligned} \quad (71)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow & ((ap (c_2Efcp_2Edimindex A_{27a}) \\ & (c_2Ebool_2Ethe_value A_{27a})) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap (c_2Epred_set_2EFINITE A_{27a}) (c_2Epred_set_2EUNIV A_{27a}))) (ap (c_2Epred_set_2ECARD A_{27a}) (c_2Epred_set_2EUNIV A_{27a}))) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\ & c_2Earithmetic_2EZERO)))))) \end{aligned} \quad (72)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow & (\forall V0n \in ty_2Enum_2Enum. (\\ & \forall V1f \in (A_{27a}^{ty_2Enum_2Enum}). ((p (ap (ap c_2Eprim_rec_2E3C c_2Enum_2E0) V0n)) \Rightarrow ((ap (c_2Elist_2EHD A_{27a}) (ap (ap (c_2Elist_2EGENLIST A_{27a}) V1f) V0n)) = (ap V1f c_2Enum_2E0)))))) \end{aligned} \quad (73)$$

Assume the following.

$((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2EiZ (ap (ap c_2Earithmetic_2E_2B V2n) V3m))))))) \wedge ((\forall V4n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum.((\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A V6n) V7m))))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V10n) V11m))))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP V14n) c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))) \wedge ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP (ap c_2Earithmetic_2ENUMERAL V15n)) (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2EEEXP V15n) V16m))))))) \wedge (((ap c_2Enum_2ESUC c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \wedge ((\forall V17n \in ty_2Enum_2Enum.((ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2ESUC V17n))))))) \wedge (((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Eprim_rec_2EPRE V18n))))))) \wedge ((\forall V19n \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum.((\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m))))))) \wedge ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V23n)) \Leftrightarrow False))) \wedge ((\forall V24n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V24n)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) V24n))))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V25n)) (ap c_2Earithmetic_2ENUMERAL V26m)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V25n) V26m))))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V27n)) \Leftrightarrow False))) \wedge ((\forall V28n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL V28n)) c_2Enum_2E0) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) V28n))))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.((\forall V30m \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V29n)) (ap c_2Earithmetic_2ENUMERAL V30m)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V29n))))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V31n)) \Leftrightarrow True))) \wedge ((\forall V32n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C c_2Enum_2E0) V32n)) \Leftrightarrow False))) \wedge ((\forall V33n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V33n)) \Leftrightarrow True))) \wedge ((\forall V34n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V34n)) \Leftrightarrow False)))$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D c_2Earithmetic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) c_2Earithmetic_2EZERO) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2EBIT2 V0n)) c_2Earithmetic_2EZERO) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n))) \wedge ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))))))))))) \\
\end{aligned} \tag{75}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\
& (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN \\
& A_27a) V2x) V0s) \Leftrightarrow (p (ap (ap (c_2Ebool_2EIN A_27a) V2x) V1t))))))) \\
\end{aligned} \tag{76}$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. (\neg(p (ap (ap \\
(c_2Ebool_2EIN A_27a) V0x) (c_2Epred_set_2EEMPTY A_27a)))))) \tag{77}$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. (p (ap (ap (c_2Ebool_2EIN \\
A_27a) V0x) (c_2Epred_set_2EUNIV A_27a)))) \tag{78}$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\neg((c_2Epred_set_2EUNIV A_27a) = \\
(c_2Epred_set_2EEMPTY A_27a))) \tag{79}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\
& A_27a. (\forall V2s \in (2^{A_27a}). ((p (ap (ap (c_2Ebool_2EIN A_27a) \\
& V0x) (ap (ap (c_2Epred_set_2EINSERT A_27a) V1y) V2s))) \Leftrightarrow ((V0x = \\
& V1y) \vee (p (ap (ap (c_2Ebool_2EIN A_27a) V0x) V2s))))))) \\
\end{aligned} \tag{80}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1s \in \\
& (2^{A_27a}). (\neg((ap (ap (c_2Epred_set_2EINSERT A_27a) V0x) V1s) = \\
& (c_2Epred_set_2EEMPTY A_27a)))))) \\
\end{aligned} \tag{81}$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty A_{27a} \Rightarrow & (\forall V0x \in A_{27a}.(\forall V1y \in \\ A_{27a}.((p(ap(ap(c_2Ebool_2EIN A_{27a}) V0x) (ap(ap(c_2Epred_set_2EINSERT \\ A_{27a}) V1y) (c_2Epred_set_2EEMPTY A_{27a})))) \Leftrightarrow (V0x = V1y)))) \\ (82) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty A_{27a} \Rightarrow & \forall A_{27b}.nonempty A_{27b} \Rightarrow \\ & (\forall V0y \in A_{27b}.(\forall V1s \in (2^{A_{27a}}).(\forall V2f \in (A_{27b}^{A_{27a}}). \\ ((p(ap(ap(c_2Ebool_2EIN A_{27b}) V0y) (ap(ap(c_2Epred_set_2EIMAGE \\ A_{27a} A_{27b}) V2f) V1s))) \Leftrightarrow (\exists V3x \in A_{27a}.((V0y = (ap V2f V3x)) \wedge \\ (p(ap(ap(c_2Ebool_2EIN A_{27a}) V3x) V1s))))))) \\ (83) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty A_{27a} \Rightarrow & \forall A_{27b}.nonempty A_{27b} \Rightarrow \\ & (\forall V0f \in (A_{27b}^{A_{27a}}).((ap(ap(c_2Epred_set_2EIMAGE A_{27a} \\ A_{27b}) V0f) (c_2Epred_set_2EEMPTY A_{27a})) = (c_2Epred_set_2EEMPTY \\ A_{27b}))) \\ (84) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty A_{27a} \Rightarrow & \forall A_{27b}.nonempty A_{27b} \Rightarrow \\ & (\forall V0f \in (A_{27b}^{A_{27a}}).(\forall V1x \in A_{27a}.(\forall V2s \in (\\ 2^{A_{27a}}).((ap(ap(c_2Epred_set_2EIMAGE A_{27a} A_{27b}) V0f) (ap \\ (ap(c_2Epred_set_2EINSERT A_{27a}) V1x) V2s)) = (ap(ap(c_2Epred_set_2EINSERT \\ A_{27b}) (ap V0f V1x)) (ap(ap(c_2Epred_set_2EIMAGE A_{27a} A_{27b}) \\ V0f) V2s))))))) \\ (85) \end{aligned}$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (p(ap(c_2Epred_set_2EFINITE \\ A_{27a}) (c_2Epred_set_2EEMPTY A_{27a}))) \quad (86)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty A_{27a} \Rightarrow & (\forall V0P \in (2^{(2^{A_{27a}})}).((\\ (p(ap V0P (c_2Epred_set_2EEMPTY A_{27a}))) \wedge (\forall V1s \in (2^{A_{27a}}). \\ (((p(ap(c_2Epred_set_2EFINITE A_{27a}) V1s)) \wedge (p(ap V0P V1s))) \Rightarrow \\ (\forall V2e \in A_{27a}.((\neg(p(ap(ap(c_2Ebool_2EIN A_{27a}) V2e) V1s))) \Rightarrow \\ (p(ap V0P (ap(ap(c_2Epred_set_2EINSERT A_{27a}) V2e) V1s))))))) \Rightarrow \\ (\forall V3s \in (2^{A_{27a}}).((p(ap(c_2Epred_set_2EFINITE A_{27a}) \\ V3s)) \Rightarrow (p(ap V0P V3s))))))) \\ (87) \end{aligned}$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0x \in A_{27a}.(\forall V1s \in (2^{A_{27a}}).((p (ap (c_2Epred_set_2EFINITE A_{27a}) (ap (ap (c_2Epred_set_2EINSERT A_{27a}) V0x) V1s))) \Leftrightarrow (p (ap (c_2Epred_set_2EFINITE A_{27a}) V1s))))) \quad (88)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0x \in A_{27a}.(p (ap (c_2Epred_set_2EFINITE A_{27a}) (ap (ap (c_2Epred_set_2EINSERT A_{27a}) V0x) (c_2Epred_set_2EEMPTY A_{27a})))) \quad (89)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty A_{27a} \Rightarrow & \forall A_{27b}.nonempty A_{27b} \Rightarrow \\ & (\forall V0s \in (2^{A_{27a}}).((p (ap (c_2Epred_set_2EFINITE A_{27a}) \\ & V0s)) \Rightarrow (\forall V1f \in (A_{27b}^{A_{27a}}).(p (ap (c_2Epred_set_2EFINITE \\ & A_{27b}) (ap (ap (c_2Epred_set_2EIMAGE A_{27a} A_{27b}) V1f) V0s)))))) \end{aligned} \quad (90)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow ((ap (c_2Epred_set_2ECARD A_{27a}) \quad (c_2Epred_set_2EEMPTY A_{27a})) = c_2Enum_2E0) \quad (91)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty A_{27a} \Rightarrow & (\forall V0s \in (2^{A_{27a}}).((p (ap \\ & (c_2Epred_set_2EFINITE A_{27a}) V0s)) \Rightarrow (\forall V1x \in A_{27a}.((\\ & ap (c_2Epred_set_2ECARD A_{27a}) (ap (ap (c_2Epred_set_2EINSERT \\ & A_{27a}) V1x) V0s)) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) \\ & (ap (ap (c_2Ebool_2EIN A_{27a}) V1x) V0s)) (ap (c_2Epred_set_2ECARD \\ & A_{27a}) V0s)) (ap c_2Enum_2ESUC (ap (c_2Epred_set_2ECARD A_{27a}) \\ & V0s))))))) \end{aligned} \quad (92)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty A_{27a} \Rightarrow & (\forall V0x \in A_{27a}.((ap (c_2Epred_set_2ECARD \\ & A_{27a}) (ap (ap (c_2Epred_set_2EINSERT A_{27a}) V0x) (c_2Epred_set_2EEMPTY \\ & A_{27a}))) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\ & c_2Earithmetic_2EZERO)))) \end{aligned} \quad (93)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (94)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (95)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (96)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (97)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (98)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V1q) \vee (p V2r)) \vee (\neg(p V1q)) \vee ((\neg(p V2r) \vee (\neg(p V0p)) \vee ((\neg(p V1q) \vee (\neg(p V0p)))))))))))) \end{aligned} \quad (99)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q) \vee (p V2r))) \wedge (((p V1q) \vee (\neg(p V0p)) \wedge ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (100)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r) \vee ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (101)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \wedge ((p V2r) \vee ((\neg(p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (102)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q) \vee (\neg(p V0p))))))) \quad (103)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (104)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (105)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p))))) \quad (106)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q))))) \quad (107)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (108)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow (\\ & \forall V0v \in (ty_2Efcp_2Ecart\ A_27a\ A_27b).((ap\ (c_2Efcp_2EFCP_HD \\ & A_27a\ A_27b)\ V0v) = (ap\ (c_2Elist_2EHD\ A_27a)\ (ap\ (c_2Efcp_2EV2L \\ & A_27a\ A_27b)\ V0v)))) \end{aligned}$$