

thm_2EfcP_2EFCP__MAP (TMR4Uw6uBiDAwDpP6X5HY7SaNzBEWBALzF)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2E$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_2E1$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2EF$ to be $(ap (c_2Ebool_2E_2E1 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2E7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2E1$

Definition 7 We define $c_2Ebool_2E_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_2E1 2) (\lambda V2t \in 2.V2t$

Let $ty_2EfcP_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2EfcP_2Efinite_image A0) \quad (1)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (2)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2Ethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (3)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (4)$$

Let $c_2EfcP_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2EfcP_2Edimindex A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (5)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (6)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (7)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (8)$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num\ ($

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x. x \in A \wedge p\ x) \text{ of type } \iota \Rightarrow \iota).$

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40\ ($

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 12 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ (ap\ c_2Ebool_2E_2F_5C\ ($

Definition 13 We define $c_2Efcp_2Efinite_index$ to be $\lambda A_27a : \iota. (ap\ (c_2Emin_2E_40\ (A_27a^{ty_2Enum_2Enum})$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Efcp_2Ecart\ A0\ A1) \quad (9)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Efcp_2Edest_cart \\ & A_27a\ A_27b \in ((A_27a^{(ty_2Efcp_2Efinite_image\ A_27b)})^{(ty_2Efcp_2Ecart\ A_27a\ A_27b)}) \end{aligned} \quad (10)$$

Definition 14 We define $c_2Efcp_2Efcp_index$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in (ty_2Efcp_2Ecart\ A_27a\ A_27b).$

Definition 15 We define c_2Efcp_2EFCP to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0g \in (A_27a^{ty_2Enum_2Enum}). (ap\ ($

Definition 16 We define $c_2Efcp_2EFCP_MAP$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27a}).$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (11)$$

Let $c_2Elist_2EGENLIST : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2EGENLIST\ A_27a \in (((ty_2Elist_2Elist \\ & A_27a)^{ty_2Enum_2Enum})^{(A_27a^{ty_2Enum_2Enum})}) \end{aligned} \quad (12)$$

Definition 17 We define c_2Efc_2EV2L to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0v \in (ty_2Efc_2Ecart\ A_27a\ A_27b) \rightarrow$

Let $c_2Elist_2EEL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2EEL\ A_27a \in ((A_27a^{(ty_2Elist_2Elist\ A_27a)})^{ty_2Enum_2Enum}) \quad (13)$$

Definition 18 We define c_2Efc_2EL2V to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0L \in (ty_2Elist_2Elist\ A_27a) \rightarrow$

Definition 19 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}) \rightarrow$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Elist_2EMAP \\ & A_27a\ A_27b \in (((ty_2Elist_2Elist\ A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{(A_27b^{A_27a})}) \end{aligned} \quad (14)$$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p\ V0t) \Leftrightarrow (p\ V1x))) \Leftrightarrow (p\ V0t))) \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (18)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p\ V0t)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in \\ & 2.(((p \ V0x) \Leftrightarrow (p \ V1x_{27})) \wedge ((p \ V1x_{27}) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_{27})))))) \Rightarrow \\ & (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_{27}) \Rightarrow (p \ V3y_{27})))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}.nonempty \ A_{27a} \Rightarrow \forall A_{27b}.nonempty \ A_{27b} \Rightarrow \forall A_{27c}. \\ & nonempty \ A_{27c} \Rightarrow (\forall V0f \in (A_{27b}^{A_{27a}}).(\forall V1g \in (A_{27a}^{A_{27c}}). \\ & (\forall V2x \in A_{27c}.((ap \ (ap \ (ap \ (c_2Ecombin_2Eo \ A_{27c} \ A_{27b} \ A_{27a}) \\ & \ V0f) \ V1g) \ V2x) = (ap \ V0f \ (ap \ V1g \ V2x)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}.nonempty \ A_{27a} \Rightarrow \forall A_{27b}.nonempty \ A_{27b} \Rightarrow (\\ & \forall V0x \in (ty_2Efc_2Ecart \ A_{27a} \ A_{27b}).(\forall V1y \in (ty_2Efc_2Ecart \\ & \ A_{27a} \ A_{27b}).((V0x = V1y) \Leftrightarrow (\forall V2i \in ty_2Enum_2Enum.((p \ (ap \\ & \ (ap \ c_2Eprim_rec_2E_3C \ V2i) \ (ap \ (c_2Efc_2Edimindex \ A_{27b}) \ (\\ & \ c_2Ebool_2Ethe_value \ A_{27b})))) \Rightarrow ((ap \ (ap \ (c_2Efc_2Efc_index \\ & \ A_{27a} \ A_{27b}) \ V0x) \ V2i) = (ap \ (ap \ (c_2Efc_2Efc_index \ A_{27a} \ A_{27b}) \\ & \ V1y) \ V2i)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}.nonempty \ A_{27a} \Rightarrow \forall A_{27b}.nonempty \ A_{27b} \Rightarrow (\\ & \forall V0g \in (A_{27a}^{ty_2Enum_2Enum}).(\forall V1i \in ty_2Enum_2Enum. \\ & ((p \ (ap \ (ap \ c_2Eprim_rec_2E_3C \ V1i) \ (ap \ (c_2Efc_2Edimindex \ A_{27b}) \\ & \ (c_2Ebool_2Ethe_value \ A_{27b})))) \Rightarrow ((ap \ (ap \ (c_2Efc_2Efc_index \\ & \ A_{27a} \ A_{27b}) \ (ap \ (c_2Efc_2EFCP \ A_{27a} \ A_{27b}) \ V0g)) \ V1i) = (ap \ V0g \\ & \ V1i)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}.nonempty \ A_{27a} \Rightarrow \forall A_{27b}.nonempty \ A_{27b} \Rightarrow (\\ & \forall V0f \in (A_{27b}^{A_{27a}}).(\forall V1g \in (A_{27a}^{ty_2Enum_2Enum}). \\ & (\forall V2n \in ty_2Enum_2Enum.((ap \ (ap \ (c_2Elist_2EMAP \ A_{27a} \ A_{27b}) \\ & \ V0f) \ (ap \ (ap \ (c_2Elist_2EGENLIST \ A_{27a}) \ V1g) \ V2n)) = (ap \ (ap \ (c_2Elist_2EGENLIST \\ & \ A_{27b}) \ (ap \ (ap \ (c_2Ecombin_2Eo \ ty_2Enum_2Enum \ A_{27b} \ A_{27a}) \ V0f) \\ & \ V1g)) \ V2n)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}.nonempty \ A_{27a} \Rightarrow (\forall V0f \in (A_{27a}^{ty_2Enum_2Enum}). \\ & (\forall V1n \in ty_2Enum_2Enum.(\forall V2x \in ty_2Enum_2Enum.(\\ & (p \ (ap \ (ap \ c_2Eprim_rec_2E_3C \ V2x) \ V1n)) \Rightarrow ((ap \ (ap \ (c_2Elist_2EEL \\ & \ A_{27a}) \ V2x) \ (ap \ (ap \ (c_2Elist_2EGENLIST \ A_{27a}) \ V0f) \ V1n)) = (ap \ V0f \\ & \ V2x)))))) \end{aligned} \quad (27)$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1v \in (ty_2Efc_2Ecart \\
& A_27a\ A_27c).((ap\ (ap\ (c_2Efc_2EFC_MAP\ A_27a\ A_27b\ A_27c)\ V0f) \\
& V1v) = (ap\ (c_2Efc_2EL2V\ A_27b\ A_27c)\ (ap\ (ap\ (c_2Elist_2EMAP\ A_27a \\
& A_27b)\ V0f)\ (ap\ (c_2Efc_2EV2L\ A_27a\ A_27c)\ V1v))))))
\end{aligned}$$