thm_2Efcp_2EFCP__UPDATE__COMMUTES (TMZbQYjsczq4HSWQtzHMPzXZYUn5ZdiANWj)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0 x \in 2.V0 x))\ (\lambda V1 x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0 P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0 t1 \in 2.(\lambda V1 t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2 t \in 2$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0 t \in 2.V0 t))$.

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0 t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0 t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0 t1 \in 2.(\lambda V1 t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2 t \in 2$

Let $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \qquad (1)$$

Let $c\_2Ebool\_2Ethe\_\_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_\_value\ A\_27a \in (\atop ty\_2Ebool\_2Eitself\ A\_27a) \qquad (2)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \qquad (3)$$

Let $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efcp\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \qquad (4)$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \qquad (5)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \qquad (6)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (7)$$

Definition 9 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\textbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \textbf{then}\ (the\ (\lambda x.x \in A \wedge$ *of type* $\iota \Rightarrow \iota$.

Definition 11 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ $c_2Emin_2E_40

Definition 12 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Let $ty\_2Efcp\_2Efinite\_\_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Efinite\_\_image\ A0) \qquad (8)$$

Definition 13 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ $c_2Ebool_2E_2F_5C

Definition 14 *We define* c_2Efcp_2Efinite_index *to be* $\lambda A\_27a : \iota.(ap\ ($c_2Emin_2E_40 $(A\_27a^{ty\_2Enum\_2Enu}$

Let $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcp\_2Ecart\ A0\ A1) \qquad (9)$$

Let $c\_2Efcp\_2Edest\_\_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efcp\_2Edest\_\_cart\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_\_image\ A\_27b)})^{(ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b)}) \qquad (10)$$

Definition 15 *We define* c_2Efcp_2Efcp_index *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart\ A\_27$

Definition 16 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 17 *We define* c_2Efcp_2EFCP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap\ ($

Definition 18 *We define* c_2Efcp_2E_3A_2B *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0a \in ty\_2Enum\_2Enum.\lambda V1b \in$

Assume the following.

$$True \tag{11}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{13}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \tag{14}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t))))))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{19}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{20}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{21}$$

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Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.$$
$$(\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a.$$
$$(\forall V5y\_27 \in A\_27a.((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge$$
$$((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (\textsf{c\_2Ebool\_2ECOND}\ A\_27a)$$
$$V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\textsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)$$
$$V5y\_27)))))))))$$

$$(22)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow ((\forall V0t1 \in A\_27a.(\forall V1t2 \in$$
$$A\_27a.((ap\ (ap\ (ap\ (\textsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \textsf{c\_2Ebool\_2ET})\ V0t1)$$
$$V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a.(\forall V3t2 \in A\_27a.((ap$$
$$(ap\ (ap\ (\textsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \textsf{c\_2Ebool\_2EF})\ V2t1)\ V3t2) = V3t2))))$$

$$(23)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow ($$
$$\forall V0x \in (ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b).(\forall V1y \in (ty\_2Efcp\_2Ecart$$
$$A\_27a\ A\_27b).((V0x = V1y) \Leftrightarrow (\forall V2i \in ty\_2Enum\_2Enum.((p\ (ap$$
$$(ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ V2i)\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27b)\ ($$
$$c\_2Ebool\_2Ethe\_\_value\ A\_27b)))) \Rightarrow ((ap\ (ap\ (\textsf{c\_2Efcp\_2Efcp\_\_index}$$
$$A\_27a\ A\_27b)\ V0x)\ V2i) = (ap\ (ap\ (\textsf{c\_2Efcp\_2Efcp\_\_index}\ A\_27a\ A\_27b)$$
$$V1y)\ V2i)))))))$$

$$(24)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow ($$
$$\forall V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(\forall V1i \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1i)\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27b)$$
$$(c\_2Ebool\_2Ethe\_\_value\ A\_27b)))) \Rightarrow ((ap\ (ap\ (\textsf{c\_2Efcp\_2Efcp\_\_index}$$
$$A\_27a\ A\_27b)\ (ap\ (\textsf{c\_2Efcp\_2EFCP}\ A\_27a\ A\_27b)\ V0g))\ V1i) = (ap\ V0g$$
$$V1i)))))$$

$$(25)$$

Theorem 1

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow ($$
$$\forall V0m \in (ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b).(\forall V1a \in ty\_2Enum\_2Enum.$$
$$(\forall V2b \in ty\_2Enum\_2Enum.(\forall V3c \in A\_27a.(\forall V4d \in$$
$$A\_27a.((\neg(V1a = V2b)) \Rightarrow ((ap\ (ap\ (ap\ (\textsf{c\_2Efcp\_2E\_3A\_2B}\ A\_27a\ A\_27b)$$
$$V1a)\ V3c)\ (ap\ (ap\ (ap\ (\textsf{c\_2Efcp\_2E\_3A\_2B}\ A\_27a\ A\_27b)\ V2b)\ V4d)\ V0m)) =$$
$$(ap\ (ap\ (ap\ (\textsf{c\_2Efcp\_2E\_3A\_2B}\ A\_27a\ A\_27b)\ V2b)\ V4d)\ (ap\ (ap\ (ap\ ($$
$$\textsf{c\_2Efcp\_2E\_3A\_2B}\ A\_27a\ A\_27b)\ V1a)\ V3c)\ V0m)))))))))$$