

thm_2EfcP_2EFCP__UPDATE__IMP__ID
(TMJmX8U9uiPzpbimwukrRPaGJJSrKenFUuX)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_7E$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (1)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2Ethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (2)$$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty ty_2Eenum_2Eenum \quad (3)$$

Let $c_2EfcP_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2EfcP_2Edimindex A_27a \in (ty_2Eenum_2Eenum^{(ty_2Ebool_2Eitself A_27a)}) \quad (4)$$

Let $c_2Eenum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Eenum_2EREP_num \in (\omega^{ty_2Eenum_2Eenum}) \quad (5)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{omega}) \quad (6)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (7)$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num)$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge p\ x))$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a\ P))))$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Efcf_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Efcf_2Efinite_image\ A0) \quad (8)$$

Definition 12 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ c_2Ebool_2E_2F_5C\ A_27a\ P))))$

Definition 13 We define $c_2Efcf_2Efinite_index$ to be $\lambda A_27a : \iota.(ap\ (c_2Emin_2E_40\ A_27a\ (ty_2Enum_2Enum)))$

Let $ty_2Efcf_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efcf_2Ecart\ A0\ A1) \quad (9)$$

Let $c_2Efcf_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efcf_2Edest_cart\ A_27a\ A_27b \in ((A_27a\ (ty_2Efcf_2Efinite_image\ A_27b))\ (ty_2Efcf_2Ecart\ A_27a\ A_27b)) \quad (10)$$

Definition 14 We define $c_2Efcf_2Efcf_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcf_2Ecart\ A_27a\ A_27b)$

Definition 15 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.))$

Definition 16 We define c_2Efcf_2EFCF to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a\ ty_2Enum_2Enum)).(ap\ (c_2Emin_2E_40\ A_27a\ (ty_2Enum_2Enum)))$

Definition 17 We define $c_2Efcf_2E_3A_2B$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0a \in ty_2Enum_2Enum.\lambda V1b \in ty_2Enum_2Enum$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0b \in 2. (\forall V1t \in A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V0b)\ V1t)\ V1t) = V1t))) \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2) \Rightarrow (p\ V2t3)) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (16)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27)\ V5y_27)))))) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0x \in (ty_2EfcP_2Ecart\ A_27a\ A_27b). (\forall V1y \in (ty_2EfcP_2Ecart\ A_27a\ A_27b). ((V0x = V1y) \Leftrightarrow (\forall V2i \in ty_2Enum_2Enum. ((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V2i)\ (ap\ (c_2EfcP_2Edimindex\ A_27b)\ (c_2Ebool_2Ethe_value\ A_27b)))))) \Rightarrow ((ap\ (ap\ (c_2EfcP_2EfcP_index\ A_27a\ A_27b)\ V0x)\ V2i) = (ap\ (ap\ (c_2EfcP_2EfcP_index\ A_27a\ A_27b)\ V1y)\ V2i)))))) \quad (19)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0g \in (A_27a^{ty_2Enum_2Enum}). (\forall V1i \in ty_2Enum_2Enum. \\
& \quad ((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V1i)\ (ap\ (c_2Efc_2Edimindex\ A_27b) \\
& \quad (c_2Ebool_2Ethe_value\ A_27b)))) \Rightarrow ((ap\ (ap\ (c_2Efc_2Efc_index \\
& \quad A_27a\ A_27b)\ (ap\ (c_2Efc_2EFCP\ A_27a\ A_27b)\ V0g))\ V1i) = (ap\ V0g \\
& \quad V1i))))))
\end{aligned} \tag{20}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0m \in (ty_2Efc_2Ecart\ A_27a\ A_27b). (\forall V1a \in ty_2Enum_2Enum. \\
& \quad (\forall V2v \in A_27a. (((ap\ (ap\ (c_2Efc_2Efc_index\ A_27a\ A_27b) \\
& \quad V0m)\ V1a) = V2v) \Rightarrow ((ap\ (ap\ (ap\ (c_2Efc_2E_3A_2B\ A_27a\ A_27b)\ V1a) \\
& \quad V2v)\ V0m) = V0m))))))
\end{aligned}$$