

thm_2EfcP_2ELENGTH_V2L (TM- cAA8sc4o38L6qoPpHXkwMLi4oDBRwhRN2)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (1)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2Ethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (2)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (3)$$

Let $c_2EfcP_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2EfcP_2Edimindex A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (4)$$

Let $ty_2EfcP_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2EfcP_2Efinite_image A0) \quad (5)$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2)) (\lambda V0t \in 2.V0t)$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2)) (\lambda V2t \in 2.V2t))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (6)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (7)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (8)$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. P\ x) \text{ then } (the\ (\lambda x. x \in A \wedge P\ x))$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ (ap\ c_2Ebool_2E_2F_5C$

Definition 13 We define $c_2Efcp_2Efinite_index$ to be $\lambda A_27a : \iota. (ap\ (c_2Emin_2E_40\ (A_27a^{ty_2Enum_2Enum}$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Efcp_2Ecart\ A0\ A1) \quad (9)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Efcp_2Edest_cart\ A_27a\ A_27b \in ((A_27a^{(ty_2Efcp_2Efinite_image\ A_27b)})^{(ty_2Efcp_2Ecart\ A_27a\ A_27b)}) \quad (10)$$

Definition 14 We define $c_2Efcp_2Efcp_index$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in (ty_2Efcp_2Ecart\ A_27a\ A_27b)$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (11)$$

Let $c_2Elist_2EGENLIST : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2EGENLIST\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{ty_2Enum_2Enum})^{(A_27a^{ty_2Enum_2Enum})}) \quad (12)$$

Definition 15 We define c_2Efcp_2EV2L to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0v \in (ty_2Efcp_2Ecart\ A_27a\ A_27b)$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \quad (13)$$

Assume the following.

$$True \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (15)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in (A_27a^{ty_2Enum_2Enum}). \\ & (\forall V1n \in ty_2Enum_2Enum. ((ap\ (c_2Elist_2ELENGTH\ A_27a) \\ & (ap\ (ap\ (c_2Elist_2EGENLIST\ A_27a)\ V0f)\ V1n)) = V1n))) \end{aligned} \quad (16)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0v \in (ty_2EfcP_2Ecart\ A_27a\ A_27b). ((ap\ (c_2Elist_2ELENGTH \\ & A_27a)\ (ap\ (c_2EfcP_2EV2L\ A_27a\ A_27b)\ V0v)) = (ap\ (c_2EfcP_2Edimindex \\ & A_27b)\ (c_2Ebool_2Ethe_value\ A_27b)))) \end{aligned}$$