

# thm\_2Efinite\_map\_2EDOMSUB\_FUPDATE\_NEQ (TMGVKgDtSdV4FBS46bH4kJcaN5cjaKfihN)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

**Definition 6** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 7** We define  $c\_2Eone\_2Eone$  to be  $(ap (c\_2Emin\_2E\_40 ty\_2Eone\_2Eone) (\lambda V0x \in ty\_2Eone\_2Eone.V0x))$

**Definition 8** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F))$

**Definition 9** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \tag{3}$$

**Definition 10** We define  $c\_Esum\_2EINR$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27b. (ap (c\_Esum\_2EABS$

Let  $ty\_2Efinite\_map\_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Efinite\_map\_2Efmap A0 A1) \quad (4)$$

Let  $c\_2Efinite\_map\_2Efmap\_ABS : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Efinite\_map\_2Efmap\_ABS A\_27a A\_27b \in ((ty\_2Efinite\_map\_2Efmap A\_27a A\_27b)^{(ty\_2Esum\_2Esum A\_27b ty\_2Eone\_2Eone)^{A\_27a}}) \quad (5)$$

**Definition 11** We define  $c\_2Efinite\_map\_2EFEMPTY$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (ap (c\_2Efinite\_map\_2E$

Let  $c\_2Efinite\_map\_2Efmap\_REP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Efinite\_map\_2Efmap\_REP A\_27a A\_27b \in (((ty\_2Esum\_2Esum A\_27b ty\_2Eone\_2Eone)^{A\_27a})^{(ty\_2Efinite\_map\_2Efmap A\_27a A\_27b)}) \quad (6)$$

Let  $c\_2Esum\_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Esum\_2EOUTL A\_27a A\_27b \in (A\_27a^{(ty\_2Esum\_2Esum A\_27a A\_27b)}) \quad (7)$$

**Definition 12** We define  $c\_2Efinite\_map\_2EFAPPLY$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (ty\_2Efinite\_map$

Let  $c\_2Esum\_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Esum\_2EISL A\_27a A\_27b \in (2^{(ty\_2Esum\_2Esum A\_27a A\_27b)}) \quad (8)$$

**Definition 13** We define  $c\_2Efinite\_map\_2EFDOM$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (ty\_2Efinite\_map$

**Definition 14** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (9)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (10)$$

**Definition 15** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap (c\_2$

Let  $c\_2Efinite\_map\_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_map\_2EFUPDATE \\ & A\_27a\ A\_27b \in (((ty\_2Efinite\_map\_2Efmap\ A\_27a\ A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})^{(ty\_2Efinite\_map)}} \end{aligned} \quad (11)$$

Let  $c\_2Efinite\_map\_2EDRESTRICT : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_map\_2EDRESTRICT \\ & A\_27a\ A\_27b \in (((ty\_2Efinite\_map\_2Efmap\ A\_27a\ A\_27b)^{(2^{A-27a})})^{(ty\_2Efinite\_map\_2Efmap\ A\_27a\ A\_27b)}} \end{aligned} \quad (12)$$

**Definition 16** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 17** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. (\lambda V1f \in (2^{A-27a}). (ap\ V1f\ V0x)))$

**Definition 18** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2. (ap\ V2t\ V1t2))))))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ & A\_27a\ A\_27b \in ((2^{A-27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A-27b})}} \end{aligned} \quad (13)$$

**Definition 19** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. \lambda V1s \in (2^{A-27a}). (ap\ (c\_2Ebool\_2E\_21\ 2)\ V1s))$

**Definition 20** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a.c\_2Ebool\_2E2ET)$ .

**Definition 21** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap\ (c\_2Ebool\_2E\_21\ 2)\ V1t))$

**Definition 22** We define  $c\_2Epred\_set\_2Ecompl$  to be  $\lambda A\_27a : \iota. \lambda V0P \in (2^{A-27a}). (ap\ (ap\ (c\_2Epred\_set\_2EUNIV)\ V0P))$

**Definition 23** We define  $c\_2Efinite\_map\_2Efdomsb$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0fm \in (ty\_2Efinite\_map\_2Efdomsb\ A\_27a\ A\_27b)$

**Definition 24** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap\ (c\_2Ebool\_2E\_21\ 2)\ V1t))$

Assume the following.

$$True \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (15)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\ & A\_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t))))))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{23}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\
& (p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in \\
& 2.(((p \ V0x) \Leftrightarrow (p \ V1x\_27)) \wedge ((p \ V1x\_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y\_27)))) \Rightarrow \\
& (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x\_27) \Rightarrow (p \ V3y\_27))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\
& (\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a. \\
& (\forall V5y\_27 \in A\_27a.(((p \ V0P) \Leftrightarrow (p \ V1Q)) \wedge (((p \ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\
& ((\neg(p \ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \\
& V0P) \ V2x) \ V4y) = (ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \ V1Q) \ V3x\_27) \\
& V5y\_27)))))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ & A\_27a. ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a. (\forall V3t2 \in A\_27a. ((ap \\ & (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF)\ V2t1)\ V3t2) = V3t2)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0f \in (ty\_2Efinite\_map\_2E fmap\ A\_27a\ A\_27b). (\forall V1a \in \\ & A\_27a. (\forall V2b \in A\_27b. ((ap\ (c\_2Efinite\_map\_2EFDOM\ A\_27a \\ & A\_27b)\ (ap\ (ap\ (c\_2Efinite\_map\_2EFUPDATE\ A\_27a\ A\_27b)\ V0f)\ (ap \\ & (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V1a)\ V2b))) = (ap\ (ap\ (c\_2Epred\_set\_2EINSERT \\ & A\_27a)\ V1a)\ (ap\ (c\_2Efinite\_map\_2EFDOM\ A\_27a\ A\_27b)\ V0f)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0f \in (ty\_2Efinite\_map\_2E fmap\ A\_27a\ A\_27b). (\forall V1a \in \\ & A\_27a. (\forall V2b \in A\_27b. (\forall V3x \in A\_27a. ((ap\ (ap\ (c\_2Efinite\_map\_2EFAPPLY \\ & A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Efinite\_map\_2EFUPDATE\ A\_27a\ A\_27b)\ V0f)\ \\ & (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V1a)\ V2b)))\ V3x) = (ap\ (ap\ (ap \\ & (c\_2Ebool\_2ECOND\ A\_27b)\ (ap\ (ap\ (c\_2Emin\_2E\_3D\ A\_27a)\ V3x)\ V1a)) \\ & V2b)\ (ap\ (ap\ (c\_2Efinite\_map\_2EFAPPLY\ A\_27a\ A\_27b)\ V0f)\ V3x)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0f \in (ty\_2Efinite\_map\_2E fmap\ A\_27a\ A\_27b). (\forall V1g \in \\ & (ty\_2Efinite\_map\_2E fmap\ A\_27a\ A\_27b). (((ap\ (c\_2Efinite\_map\_2EFDOM \\ & A\_27a\ A\_27b)\ V0f) = (ap\ (c\_2Efinite\_map\_2EFDOM\ A\_27a\ A\_27b)\ V1g))) \wedge \\ & (\forall V2x \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ (ap\ ( \\ & c\_2Efinite\_map\_2EFDOM\ A\_27a\ A\_27b)\ V0f))) \Rightarrow ((ap\ (ap\ (c\_2Efinite\_map\_2EFAPPLY \\ & A\_27a\ A\_27b)\ V0f)\ V2x) = (ap\ (ap\ (c\_2Efinite\_map\_2EFAPPLY\ A\_27a \\ & A\_27b)\ V1g)\ V2x)))))) \Leftrightarrow (V0f = V1g))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0f \in (ty\_2Efinite\_map\_2E fmap\ A\_27a\ A\_27b).(\forall V1r \in \\
& \quad (2^{A\_27a}).(((ap\ (c\_2Efinite\_map\_2EFDOM\ A\_27a\ A\_27b)\ (ap\ (ap \\
& \quad (c\_2Efinite\_map\_2EDRESTRICT\ A\_27a\ A\_27b)\ V0f)\ V1r))) = (ap\ (ap \\
& \quad (c\_2Epred\_set\_2EINTER\ A\_27a)\ (ap\ (c\_2Efinite\_map\_2EFDOM\ A\_27a \\
& \quad A\_27b)\ V0f))\ V1r)) \wedge (\forall V2x \in A\_27a.((ap\ (ap\ (c\_2Efinite\_map\_2EFAPPLY \\
& \quad A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Efinite\_map\_2EDRESTRICT\ A\_27a\ A\_27b) \\
& \quad V0f)\ V1r))\ V2x) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27b)\ (ap\ (ap\ (c\_2Ebool\_2EIN \\
& \quad A\_27a)\ V2x)\ (ap\ (ap\ (c\_2Epred\_set\_2EINTER\ A\_27a)\ (ap\ (c\_2Efinite\_map\_2EFDOM \\
& \quad A\_27a\ A\_27b)\ V0f))\ V1r)))\ (ap\ (ap\ (c\_2Efinite\_map\_2EFAPPLY\ A\_27a \\
& \quad A\_27b)\ V0f)\ V2x))\ (ap\ (ap\ (c\_2Efinite\_map\_2EFAPPLY\ A\_27a\ A\_27b) \\
& \quad (c\_2Efinite\_map\_2EFEMPTY\ A\_27a\ A\_27b))\ V2x)))))) \\
& \hspace{15em} (31)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0f \in (ty\_2Efinite\_map\_2E fmap\ A\_27a\ A\_27b).(\forall V1r \in \\
& \quad (2^{A\_27a}).(\forall V2x \in A\_27a.(\forall V3y \in A\_27b.((ap\ (ap\ (c\_2Efinite\_map\_2EDRESTRICT \\
& \quad A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Efinite\_map\_2EFUPDATE\ A\_27a\ A\_27b)\ V0f) \\
& \quad (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V2x)\ V3y)))\ V1r) = (ap\ (ap\ (ap \\
& \quad (c\_2Ebool\_2ECOND\ (ty\_2Efinite\_map\_2E fmap\ A\_27a\ A\_27b))\ (ap \\
& \quad (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V1r))\ (ap\ (ap\ (c\_2Efinite\_map\_2EFUPDATE \\
& \quad A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Efinite\_map\_2EDRESTRICT\ A\_27a\ A\_27b) \\
& \quad V0f)\ V1r))\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V2x)\ V3y)))\ (ap\ ( \\
& \quad ap\ (c\_2Efinite\_map\_2EDRESTRICT\ A\_27a\ A\_27b)\ V0f)\ V1r)))))) \\
& \hspace{15em} (32)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in \\
& \quad (2^{A\_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a.((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\
& \quad A\_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V1t)))))) \\
& \hspace{15em} (33)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\neg (p\ (ap\ (ap \\
& \quad (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ (c\_2Epred\_set\_2EEMPTY\ A\_27a)))) \\
& \hspace{15em} (34)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in \\
& \quad (2^{A\_27a}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a) \\
& \quad V2x)\ (ap\ (ap\ (c\_2Epred\_set\_2EINTER\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\
& \quad (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\
& \quad A\_27a)\ V2x)\ V1t)))))) \\
& \hspace{15em} (35)
\end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in \\ & A.27a. (\forall V2s \in (2^{A.27a}). ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a) \\ & V0x)\ (ap\ (ap\ (c.2Epred\_set.2EINSERT\ A.27a)\ V1y)\ V2s))) \Leftrightarrow ((V0x = \\ & V1y) \vee (p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V0x)\ V2s)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1s \in \\ & (2^{A.27a}). ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V0x)\ (ap\ (c.2Epred\_set.2ECOMPL \\ & A.27a)\ V1s))) \Leftrightarrow (\neg(p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V0x)\ V1s)))))) \end{aligned} \quad (37)$$

**Theorem 1**

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ & \forall V0fm \in (ty\_2Efinite\_map\_2Efmap\ A.27a\ A.27b). (\forall V1k1 \in \\ & A.27a. (\forall V2k2 \in A.27a. (\forall V3v \in A.27b. ((\neg(V1k1 = V2k2)) \Rightarrow \\ & ((ap\ (ap\ (c.2Efinite\_map\_2Efdomsb\ A.27a\ A.27b)\ (ap\ (ap\ (c.2Efinite\_map\_2EFUPDATE \\ & A.27a\ A.27b)\ V0fm)\ (ap\ (ap\ (c.2Epair\_2E\_2C\ A.27a\ A.27b)\ V1k1)\ V3v))) \\ & V2k2) = (ap\ (ap\ (c.2Efinite\_map\_2EFUPDATE\ A.27a\ A.27b)\ (ap\ (ap \\ & (c.2Efinite\_map\_2Efdomsb\ A.27a\ A.27b)\ V0fm)\ V2k2))\ (ap\ (ap\ ( \\ & c.2Epair\_2E\_2C\ A.27a\ A.27b)\ V1k1)\ V3v)))))) \end{aligned}$$