

thm_2Efinite_map_2EDRESTRICT_MAP_KEYS_IMAGE (TMX5eRCFfDyqeXrqQQxuggYGZPvn729x16U)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota).$

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V 0x \in 2. V 0x)) (\lambda V 1x \in 2. V 1x))$

Let `ty_2Efinite_map_2Efmap` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \forall A 1. \text{nonempty } A 1 \Rightarrow \text{nonempty } (\text{ty_2Efinite_map_2Efmap } A 0 \ A 1) \quad (1)$$

Let `c_2Efinite_map_2EDRESTRICT` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow \text{c_2Efinite_map_2EDRESTRICT } A. 27a \ A. 27b \in (((\text{ty_2Efinite_map_2Efmap } A. 27a \ A. 27b)^{(2^{A-27a}}))(\text{ty_2Efinite_map_2Efmap } A. 27a \ A. 27b)) \quad (2)$$

Let `c_2Efinite_map_2EMAP_KEYS` : $\iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow \forall A. 27c. \text{nonempty } A. 27c \Rightarrow \text{c_2Efinite_map_2EMAP_KEYS } A. 27a \ A. 27b \ A. 27c \in (((\text{ty_2Efinite_map_2Efmap } A. 27b \ A. 27c)^{(\text{ty_2Efinite_map_2Efmap } A. 27a \ A. 27c)})^{(A. 27b^{A-27a}})) \quad (3)$$

Let `ty_2Eone_2Eone` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Eone_2Eone} \quad (4)$$

Definition 4 We define `c_2Eone_2Eone` to be $(\text{ap } (\text{c_2Emin_2E_40 } \text{ty_2Eone_2Eone}) (\lambda V 0x \in \text{ty_2Eone_2Eone}))$

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 6 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21\ 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in 2.V2t))))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (5)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (6)$$

Definition 10 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap (c_2Esum_2EABS_sum\ A_27a\ A_27b) V0e)$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (7)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (8)$$

Definition 11 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap (c_2Eoption_2Eoption_ABS\ A_27a) (ty_2Eone_2Eone))$

Let $c_2Efinite_map_2Efmap_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2Efmap_REP\ A_27a\ A_27b \in (((ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a})^{(ty_2Efinite_map_2Efmap\ A_27a\ A_27b)}) \quad (9)$$

Let $c_2Esum_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EOUTL\ A_27a\ A_27b \in (A_27a^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \quad (10)$$

Definition 12 We define $c_2Efinite_map_2EFAPPLY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map_2EFAPPLY\ A_27a\ A_27b) V0f$

Definition 13 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_2Esum_2EABS_sum\ A_27a\ A_27b) V0e)$

Definition 14 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap (c_2Eoption_2Eoption_ABS\ A_27a) V0x)$

Let $c_2Esum_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EISL \\ A_27a\ A_27b \in (2^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \end{aligned} \quad (11)$$

Definition 15 We define $c_2Efinite_map_2EFDOM$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map$

Definition 16 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x))$

Definition 17 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 18 We define $c_2Efinite_map_2EFLOOKUP$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map$

Let $c_2Eoption_2EIS_SOME : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2EIS_SOME\ A_27a \in (2^{(ty_2Eoption_2Eoption\ A_27a)}) \quad (12)$$

Let $c_2Eoption_2EIS_NONE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2EIS_NONE\ A_27a \in (2^{(ty_2Eoption_2Eoption\ A_27a)}) \quad (13)$$

Let $c_2Eoption_2EOPTION_BIND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2EOPTION_BIND \\ A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Eoption_2Eoption\ A_27a)^{A_27b}})^{(ty_2Eoption_2Eoption\ A_27b)}) \end{aligned} \quad (14)$$

Definition 19 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 20 We define $c_2Eoption_2Esome$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (ap\ (c_2Ebool_2EC$

Definition 21 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (15)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (16)$$

Definition 22 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \end{aligned} \quad (17)$$

Definition 23 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EINTER\ V0s\ V1t))$

Definition 24 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (A_27b^{A_27a}).(ap\ (c_2Epred_set_2EIMAGE\ V0f\ V1s))$

Definition 25 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (A_27b^{A_27a}).(ap\ (c_2Epred_set_2EINJ\ V0f\ V1s))$

Definition 26 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (V0t1\ V1t2))))$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (21)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} ((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True)))) \end{aligned} \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27b^{A_27a}). ((V0f = V1g) \Leftrightarrow (\forall V2x \in A_27a. ((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V0t1)\ V1t2) = V1t2)))))) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0b \in 2. (\forall V1t \in A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V0b)\ V1t)\ V1t) = V1t))) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg(\exists V1x \in A_27a. (p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A_27a. (\neg(p\ (ap\ V0P\ V2x))))))) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\forall V3x \in A_27a. (p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A_27a. (p\ (ap\ V1Q\ V4x)))))))))) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). (((p\ V0P) \wedge (\forall V2x \in A_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x))))))) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A_27a. (p\ (ap\ V1Q\ V3x))))))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee \neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \wedge \neg(p\ V1B))))))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee ((p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C)))))) \quad (37)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (38)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow ((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((ap\ (ap\ (ap\ (c.2Ebool_2ECOND\ A_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c.2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27)\ V5y_27)))))) \quad (40)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. ((ap\ (ap\ (ap\ (c.2Ebool_2ECOND\ A_27a)\ c.2Ebool_2ET)\ V0t1)\ V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap\ (ap\ (ap\ (c.2Ebool_2ECOND\ A_27a)\ c.2Ebool_2EF)\ V2t1)\ V3t2) = V3t2)))) \quad (41)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c.nonempty\ A_27c \Rightarrow (\forall V0f \in (ty_2Efinite_map_2Efmmap\ A_27a\ A_27b). (\forall V1r \in (2^{A_27a}). (\forall V2x \in A_27c. ((ap\ (c.2Efinite_map_2EFDOM\ A_27a\ A_27b)\ (ap\ (ap\ (c.2Efinite_map_2EDRESTRICT\ A_27a\ A_27b)\ V0f)\ V1r)) = (ap\ (ap\ (c.2Epred_set_2EINTER\ A_27a)\ (ap\ (c.2Efinite_map_2EFDOM\ A_27a\ A_27b)\ V0f))\ V1r)))))) \quad (42)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f1 \in (ty_2Efinite_map_2Efmmap\ A_27a\ A_27b).(\forall V1f2 \in \\ & (ty_2Efinite_map_2Efmmap\ A_27a\ A_27b).((V0f1 = V1f2) \Leftrightarrow ((ap\ (c_2Efinite_map_2EFLOOKUP \\ & \quad A_27a\ A_27b)\ V0f1) = (ap\ (c_2Efinite_map_2EFLOOKUP\ A_27a\ A_27b) \\ & \quad \quad V1f2)))))) \end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0fm \in (ty_2Efinite_map_2Efmmap\ A_27a\ A_27b).(\forall V1s \in \\ & \quad (2^{A_27a}).(\forall V2k \in A_27a.((ap\ (ap\ (c_2Efinite_map_2EFLOOKUP \\ & \quad A_27a\ A_27b)\ (ap\ (ap\ (c_2Efinite_map_2EDRESTRICT\ A_27a\ A_27b) \\ & \quad V0fm)\ V1s))\ V2k) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption \\ & \quad A_27b))\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2k)\ V1s))\ (ap\ (ap\ (c_2Efinite_map_2EFLOOKUP \\ & \quad A_27a\ A_27b)\ V0fm)\ V2k))\ (c_2Eoption_2ENONE\ A_27b)))))) \end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1m \in (ty_2Efinite_map_2Efmmap \\ & \quad A_27a\ A_27c).(\forall V2k \in A_27b.((p\ (ap\ (ap\ (ap\ (c_2Epred_set_2EINJ \\ & \quad A_27a\ A_27b)\ V0f)\ (ap\ (c_2Efinite_map_2EFDOM\ A_27a\ A_27c)\ V1m)) \\ & \quad (c_2Epred_set_2EUNIV\ A_27b)))) \Rightarrow ((ap\ (ap\ (c_2Efinite_map_2EFLOOKUP \\ & \quad A_27b\ A_27c)\ (ap\ (ap\ (c_2Efinite_map_2EMAP_KEYS\ A_27a\ A_27b \\ & \quad A_27c)\ V0f)\ V1m))\ V2k) = (ap\ (ap\ (c_2Eoption_2EOPTION_BIND\ A_27c \\ & \quad A_27a)\ (ap\ (c_2Eoption_2Esome\ A_27a)\ (\lambda V3x \in A_27a.(ap\ (ap\ c_2Ebool_2E_2F_5C \\ & \quad (ap\ (ap\ (c_2Emin_2E_3D\ A_27b)\ V2k)\ (ap\ V0f\ V3x)))\ (ap\ (ap\ (c_2Ebool_2EIN \\ & \quad A_27a)\ V3x)\ (ap\ (c_2Efinite_map_2EFDOM\ A_27a\ A_27c)\ V1m)))))) \\ & \quad (ap\ (c_2Efinite_map_2EFLOOKUP\ A_27a\ A_27c)\ V1m)))))) \end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (ty_2Eoption_2Eoption \\ & \quad A_27a).((p\ (ap\ (c_2Eoption_2EIS_NONE\ A_27a)\ V0x) \Leftrightarrow (V0x = (c_2Eoption_2ENONE \\ & \quad \quad A_27a)))) \end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in 2. (\forall V1X \in (\\
& \quad ty_2Eoption_2Eoption\ A.27a). (\forall V2x \in A.27a. (((ap\ (ap\ (\\
& \quad \quad ap\ (c.2Ebool_2ECOND\ (ty_2Eoption_2Eoption\ A.27a))\ V0P)\ V1X)\ (\\
& \quad \quad c.2Eoption_2ENONE\ A.27a)) = (c.2Eoption_2ENONE\ A.27a)) \Leftrightarrow ((p\ V0P) \Rightarrow \\
& \quad (p\ (ap\ (c.2Eoption_2EIS_NONE\ A.27a)\ V1X)))) \wedge (((ap\ (ap\ (ap\ (c.2Ebool_2ECOND \\
& \quad \quad (ty_2Eoption_2Eoption\ A.27a))\ V0P)\ (c.2Eoption_2ENONE\ A.27a)) \\
& \quad \quad V1X) = (c.2Eoption_2ENONE\ A.27a)) \Leftrightarrow ((p\ (ap\ (c.2Eoption_2EIS_SOME \\
& \quad \quad A.27a)\ V1X)) \Rightarrow (p\ V0P))) \wedge (((ap\ (ap\ (ap\ (c.2Ebool_2ECOND\ (ty_2Eoption_2Eoption \\
& \quad \quad A.27a))\ V0P)\ V1X)\ (c.2Eoption_2ENONE\ A.27a)) = (ap\ (c.2Eoption_2ESOME \\
& \quad \quad A.27a)\ V2x)) \Leftrightarrow ((p\ V0P) \wedge (V1X = (ap\ (c.2Eoption_2ESOME\ A.27a)\ V2x)))) \wedge \\
& \quad (((ap\ (ap\ (ap\ (c.2Ebool_2ECOND\ (ty_2Eoption_2Eoption\ A.27a)) \\
& \quad \quad V0P)\ (c.2Eoption_2ENONE\ A.27a))\ V1X) = (ap\ (c.2Eoption_2ESOME \\
& \quad \quad A.27a)\ V2x)) \Leftrightarrow ((\neg(p\ V0P)) \wedge (V1X = (ap\ (c.2Eoption_2ESOME\ A.27a) \\
& \quad \quad V2x)))))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad (\forall V0f \in ((ty_2Eoption_2Eoption\ A.27a)^{A.27b}). ((ap\ (ap\ (\\
& \quad \quad c.2Eoption_2EOPTION_BIND\ A.27a\ A.27b)\ (c.2Eoption_2ENONE\ A.27b)) \\
& \quad \quad V0f) = (c.2Eoption_2ENONE\ A.27a))) \wedge (\forall V1x \in A.27b. (\forall V2f \in \\
& \quad ((ty_2Eoption_2Eoption\ A.27a)^{A.27b}). ((ap\ (ap\ (c.2Eoption_2EOPTION_BIND \\
& \quad \quad A.27a\ A.27b)\ (ap\ (c.2Eoption_2ESOME\ A.27b)\ V1x))\ V2f) = (ap\ V2f\ V1x))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in \\
& \quad (2^{(ty_2Eoption_2Eoption\ A.27a)}). (((\forall V2x \in A.27a. ((p \\
& \quad \quad (ap\ V0P\ V2x)) \Rightarrow (p\ (ap\ V1Q\ (ap\ (c.2Eoption_2ESOME\ A.27a)\ V2x)))))) \wedge \\
& \quad ((\forall V3x \in A.27a. (\neg(p\ (ap\ V0P\ V3x)))) \Rightarrow (p\ (ap\ V1Q\ (c.2Eoption_2ENONE \\
& \quad \quad A.27a)))))) \Rightarrow (p\ (ap\ V1Q\ (ap\ (c.2Eoption_2ESOME\ A.27a)\ V0P))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (p\ (ap\ (ap\ (c.2Ebool_2EIN \\
A.27a)\ V0x)\ (c.2Epred_set_2EUNIV\ A.27a)))) \tag{50}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in \\
& \quad (2^{A.27a}). (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27a) \\
& \quad \quad V2x)\ (ap\ (ap\ (c.2Epred_set_2EINTER\ A.27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\
& \quad \quad (ap\ (c.2Ebool_2EIN\ A.27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c.2Ebool_2EIN \\
& \quad \quad A.27a)\ V2x)\ V1t))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b^{A_27a}). \\ & ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\ & \quad A_27a\ A_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\ & \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s)))))) \\ & \end{aligned} \tag{52}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{53}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{54}$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \\ & \end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \\ & \end{aligned} \tag{56}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{57}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ & \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))) \\ & \end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & \quad (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\ & \quad (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))) \\ & \end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & \quad (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\ & \quad ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))) \\ & \end{aligned} \tag{60}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\neg(p V1q) \vee ((p V2r) \vee \neg(p V0p)))))))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (\neg(p V1q) \vee \neg(p V0p)))))) \quad (62)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (63)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow \neg(p V1q))) \quad (64)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V0p))) \quad (65)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V1q))) \quad (66)$$

Assume the following.

$$(\forall V0p \in 2. (\neg(\neg(p V0p))) \Rightarrow (p V0p)) \quad (67)$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow \forall A.27c. \\ & nonempty \ A.27c \Rightarrow (\forall V0f \in (A.27b^{A.27a}). (\forall V1fm \in (ty_2Efinite_map_2E fmap \\ & \ A.27a \ A.27c). (\forall V2s \in (2^{A.27a}). ((p \ (ap \ (ap \ (ap \ (c.2Epred_set_2EINJ \\ & \ A.27a \ A.27b) \ V0f) \ (c.2Epred_set_2EUNIV \ A.27a)) \ (c.2Epred_set_2EUNIV \\ & \ A.27b)))) \Rightarrow ((ap \ (ap \ (c.2Efinite_map_2EDRESTRICT \ A.27b \ A.27c) \\ & \ (ap \ (ap \ (c.2Efinite_map_2EMAP_KEYS \ A.27a \ A.27b \ A.27c) \ V0f) \ V1fm)) \\ & \ (ap \ (ap \ (c.2Epred_set_2EIMAGE \ A.27a \ A.27b) \ V0f) \ V2s)) = (ap \ (ap \\ & \ (c.2Efinite_map_2EMAP_KEYS \ A.27a \ A.27b \ A.27c) \ V0f) \ (ap \ (ap \ (\\ & \ c.2Efinite_map_2EDRESTRICT \ A.27a \ A.27c) \ V1fm) \ V2s)))))) \end{aligned}$$