

# thm\_2Efinite\_map\_2EFAPPLY\_FUPDATE\_THM (TMGKpef7e2ksyV2GJ2pzZMoN3nWFA71SzQm)

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**Definition 1** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 2** We define `c_2Emin_2E_3D` to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define `c_2Ebool_2E_2T` to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define `c_2Ebool_2E_21` to be  $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

**Definition 5** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

**Definition 6** We define `c_2Ebool_2E_2F` to be  $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 7** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

**Definition 8** We define `c_2Emin_2E_40` to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 9** We define `c_2Ebool_2ECOND` to be  $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_2Emin_2E_40 (2^{A_27a}))$

Let `ty_2Epair_2Eprod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let `c_2Epair_2EABS_prod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS\_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

**Definition 10** We define `c_2Epair_2E_2C` to be  $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Emin_2E_40 (2^{A_27a}))$

Let  $ty\_2Efinite\_map\_2E fmap : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efinite\_map\_2E\ fmap\ A0\ A1) \quad (3)$$

Let  $c\_2Efinite\_map\_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_map\_2EFUPDATE\ A\_27a\ A\_27b \in (((ty\_2Efinite\_map\_2E\ fmap\ A\_27a\ A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})^{(ty\_2Efinite\_map\_2E\ fmap\ A\_27a\ A\_27b)}) \quad (4)$$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (5)$$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (6)$$

Let  $c\_2Efinite\_map\_2E fmap\_REP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_map\_2E\ fmap\_REP\ A\_27a\ A\_27b \in (((ty\_2Esum\_2Esum\ A\_27b\ ty\_2Eone\_2Eone)^{A\_27a})^{(ty\_2Efinite\_map\_2E\ fmap\ A\_27a\ A\_27b)}) \quad (7)$$

Let  $c\_2Esum\_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EOUTL\ A\_27a\ A\_27b \in (A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)} \quad (8)$$

**Definition 11** We define  $c\_2Efinite\_map\_2EFAPPLY$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (ty\_2Efinite\_map\_2E\ fmap\_REP\ A\_27a\ A\_27b). V0f$

**Definition 12** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_7E\ V0t))$

Assume the following.

$$True \quad (9)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \vee \neg(p\ V0t))) \quad (12)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\ & p\ V0t)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ & A\_27a. (((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1)\ \\ & V1t2) = V0t1) \wedge ((ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF \\ & V0t1)\ V1t2) = V1t2)))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0f \in (ty\_2Efinite\_map\_2E fmap\ A\_27a\ A\_27b). (\forall V1x \in \\ & A\_27a. (\forall V2y \in A\_27b. ((ap\ (ap\ (c\_2Efinite\_map\_2EFAPPLY \\ & A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Efinite\_map\_2EFUPDATE\ A\_27a\ A\_27b)\ V0f) \\ & (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V1x)\ V2y))))\ V1x) = V2y)))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0f \in (ty\_2Efinite\_map\_2E fmap\ A\_27a\ A\_27b). (\forall V1a \in \\ & A\_27a. (\forall V2x \in A\_27a. (\forall V3y \in A\_27b. ((\neg(V1a = V2x)) \Rightarrow \\ & ((ap\ (ap\ (c\_2Efinite\_map\_2EFAPPLY\ A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Efinite\_map\_2EFUPDATE \\ & A\_27a\ A\_27b)\ V0f) (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V2x)\ V3y)))) \\ & V1a) = (ap\ (ap\ (c\_2Efinite\_map\_2EFAPPLY\ A\_27a\ A\_27b)\ V0f)\ V1a)))))) \end{aligned} \quad (17)$$

### Theorem 1

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0f \in (ty\_2Efinite\_map\_2E fmap\ A\_27a\ A\_27b). (\forall V1a \in \\ & A\_27a. (\forall V2b \in A\_27b. (\forall V3x \in A\_27a. ((ap\ (ap\ (c\_2Efinite\_map\_2EFAPPLY \\ & A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Efinite\_map\_2EFUPDATE\ A\_27a\ A\_27b)\ V0f) \\ & (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V1a)\ V2b))))\ V3x) = (ap\ (ap\ (ap \\ & (c\_2Ebool\_2ECOND\ A\_27b)\ (ap\ (ap\ (c\_2Emin\_2E\_3D\ A\_27a)\ V3x)\ V1a)) \\ & V2b)\ (ap\ (ap\ (c\_2Efinite\_map\_2EFAPPLY\ A\_27a\ A\_27b)\ V0f)\ V3x)))))) \end{aligned}$$