

thm_2Efinite__map_2EFDOM__DOMSUB
(TMSMGYUnScjDtrB-
HNh3UYeB5LNCAG38yYiK)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $ty_2Efinite_map_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efinite_map_2Efmap\ A0\ A1) \tag{3}$$

Let $c_2Efinite_map_2Efmap_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2Efmap_REP\ A_27a\ A_27b \in (((ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a})^{(ty_2Efinite_map_2Efmap\ A_27a\ A_27b)}) \tag{4}$$

Let $c_2Esum_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EISL\ A_27a\ A_27b \in (2^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \tag{5}$$

Definition 6 We define $c_2Efinite_map_2EFDOM$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map_2EFDOM A_27a A_27b)$.
Let $c_2Efinite_map_2EDRESTRICT : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2EDRESTRICT A_27a A_27b \in ((ty_2Efinite_map_2Efdom A_27a A_27b)^{(2^{A-27a})})^{(ty_2Efinite_map_2Efdom A_27a A_27b)} \quad (6)$$

Definition 7 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a.c_2Ebool_2EFDOM)$.

Definition 8 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A-27a}). (ap V1f V0x)))$

Definition 9 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2))))$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (7)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A-27b})})^{(2^{A-27a})} \quad (8)$$

Definition 11 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2)))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A-27a})^{(ty_2Epair_2Eprod A_27a 2)^{A-27b}}) \quad (9)$$

Definition 12 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A-27a}). (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2)))$

Definition 13 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a.c_2Ebool_2EFDOM)$.

Definition 14 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21 2))$

Definition 15 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2)))$

Definition 16 We define $c_2Epred_set_2Ecompl$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{A-27a}). (ap (ap (c_2Epred_set_2EEMPTY) (\lambda V1s \in (2^{A-27a}))) (\lambda V2t \in 2)))$

Definition 17 We define $c_2Efinite_map_2Efdomsub$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0fm \in (ty_2Efinite_map_2EFDOM A_27a A_27b)$

Definition 18 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2)))$

Definition 19 We define $c_2Epred_set_2EDELETE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1x \in A_27a. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2)))$

Assume the following.

$$True \quad (10)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (15)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c. \\ & nonempty A_27c \Rightarrow (\forall V0f \in (ty_2Efinite_map_2E fmap A_27a \\ & A_27b).(\forall V1r \in (2^{A_27a}).(\forall V2x \in A_27c.((ap (c_2Efinite_map_2EFDOM \\ & A_27a A_27b) (ap (ap (c_2Efinite_map_2EDRESTRICT A_27a A_27b) \\ & V0f) V1r)) = (ap (ap (c_2Epred_set_2EINTER A_27a) (ap (c_2Efinite_map_2EFDOM \\ & A_27a A_27b) V0f)) V1r)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1t \in (2^{A_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V2x) V0s)) \Leftrightarrow (p (ap (ap (c_2Ebool_2EIN A_27a) V2x) V1t))))))) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\neg (p (ap (ap (c_2Ebool_2EIN A_27a) V0x) (c_2Epred_set_2EEMPTY A_27a)))))) \quad (18)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in \\ (2^{A.27a}). (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a) \\ V2x)\ (ap\ (ap\ (c.2Epred_set.2EINTER\ A.27a)\ V0s)\ V1t)))) \Leftrightarrow ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V1t)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in \\ A.27a. (\forall V2s \in (2^{A.27a}). ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a) \\ V0x)\ (ap\ (ap\ (c.2Epred_set.2EINSERT\ A.27a)\ V1y)\ V2s)))) \Leftrightarrow ((V0x = \\ V1y) \vee (p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V0x)\ V2s)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1x \in \\ A.27a. (\forall V2y \in A.27a. ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V1x) \\ (ap\ (ap\ (c.2Epred_set.2EDELETE\ A.27a)\ V0s)\ V2y)))) \Leftrightarrow ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V1x)\ V0s)) \wedge \neg(V1x = V2y)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1s \in \\ (2^{A.27a}). ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V0x)\ (ap\ (c.2Epred_set.2ECOMPL \\ A.27a)\ V1s)))) \Leftrightarrow \neg(p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V0x)\ V1s)))))) \end{aligned} \quad (22)$$

Theorem 1

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ \forall V0fm \in (ty.2Efinite_map.2Efmap\ A.27a\ A.27b). (\forall V1k \in \\ A.27a. ((ap\ (c.2Efinite_map.2EFDOM\ A.27a\ A.27b)\ (ap\ (ap\ (c.2Efinite_map.2Efdomsub \\ A.27a\ A.27b)\ V0fm)\ V1k)) = (ap\ (ap\ (c.2Epred_set.2EDELETE\ A.27a) \\ (ap\ (c.2Efinite_map.2EFDOM\ A.27a\ A.27b)\ V0fm))\ V1k)))) \end{aligned}$$