thm_2Efinite__map_2EFDOM__FMERGE (TMJyJjw1LuwtncaHAU2pNSKyLtTGwSgpXDS)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj__o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c_2Emin_2E_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2EIN *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (\mathsf{c_2Emin_2E_3D}\ (2^{A_27a}$

Definition 5 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c_2Ebool_2E_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj__o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c_2Emin_2E_3D_3D_3E}\ V0t)\ \mathsf{c_2Ebool_2EF}$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c_2Ebool_2E_21}\ 2)\ (\lambda V2t \in 2$

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2ECOND *to be* $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $ty_2Efinite__map_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efinite__map_2Efmap\ A0\ A1) \tag{3}$$

Let $c_2Efinite__map_2Efmap__REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite__map_2Efmap__REP\ A_27a\ A_27b \in (((ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a})^{(ty_2Efinite__map_2Efmap\ A_27a\ A_27b)}) \tag{4}$$

Let $c_2Esum_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EOUTL\ A_27a\ A_27b \in (A_27a^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \tag{5}$$

Definition 11 *We define* c_2Efinite__map_2EFAPPLY *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite__ma$

Definition 12 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c_2Ebool_2E_21}\ 2)\ (\lambda V2t \in$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{6}$$

Let $c_2Epair_2EABS__prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS__prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})}) \tag{7}$$

Definition 13 *We define* c_2Epair_2E_2C *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2$

Let $c_2Epred__set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred__set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \tag{8}$$

Definition 14 *We define* c_2Epred__set_2EUNION *to be* $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c$

Let $c_2Efinite__map_2EFMERGE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite__map_2$$
$$A_27a\ A_27b \in ((((ty_2Efinite__map_2Efmap\ A_27b\ A_27a)^{(ty_2Efinite__map_2Efmap\ A_27b\ A_27a)})^{(ty_2Efi} \tag{9}$$

Let $c_2Esum_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EISL\ A_27a\ A_27b \in (2^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \tag{10}$$

Definition 15 *We define* c_2Efinite__map_2EFDOM *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite__map$

Assume the following.

$$True \tag{11}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{12}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0m \in ((A_27a^{A_27a})^{A_27a}).(\forall V1f \in (ty_2Efinite__map_2Efmap\ A_27b\ A_27a).(\forall V2g \in (ty_2Efinite__map_2Efmap\ A_27b\ A_27a). (((ap\ (\mathsf{c_2Efinite__map_2EFDOM}\ A_27b\ A_27a)\ (ap\ (ap\ (ap\ (c_2Efinite__map_2EFMERGE\ A_27a\ A_27b)\ V0m)\ V1f)\ V2g)) = (ap\ (ap\ (\mathsf{c_2Epred__set_2EUNION}\ A_27b)\ (ap\ (\mathsf{c_2Efinite__map_2EFDOM}\ A_27b\ A_27a)\ V1f))\ (ap\ (\mathsf{c_2Efinite__map_2EFDOM}\ A_27b\ A_27a)\ V2g)))\wedge(\forall V3x \in A_27b.((ap\ (ap\ (\mathsf{c_2Efinite__map_2EFAPPLY}\ A_27b\ A_27a)\ (ap\ (ap\ (ap\ (c_2Efinite__map_2EFMERGE\ A_27a\ A_27b)\ V0m)\ V1f)\ V2g))\ V3x) = (ap\ (ap\ (ap\ (\mathsf{c_2Ebool_2ECOND}\ A_27a)\ (ap\ \mathsf{c_2Ebool_2E_7E}\ (ap\ (ap\ (\mathsf{c_2Ebool_2EIN}\ A_27b)\ V3x)\ (ap\ (\mathsf{c_2Efinite__map_2EFDOM}\ A_27b\ A_27a)\ V1f))))\ (ap\ (ap\ (\mathsf{c_2Efinite__map_2EFAPPLY}\ A_27b\ A_27a)\ V2g)\ V3x))\ (ap\ (ap\ (ap\ (\mathsf{c_2Ebool_2ECOND}\ A_27a)\ (ap\ \mathsf{c_2Ebool_2E_7E}\ (ap\ (ap\ (\mathsf{c_2Ebool_2EIN}\ A_27b)\ V3x)\ (ap\ (\mathsf{c_2Efinite__map_2EFDOM}\ A_27b\ A_27a)\ V2g))))\ (ap\ (ap\ (\mathsf{c_2Efinite__map_2EFAPPLY}\ A_27b\ A_27a)\ V1f)\ V3x))\ (ap\ (ap\ V0m\ (ap\ (ap\ (\mathsf{c_2Efinite__map_2EFAPPLY}\ A_27b\ A_27a)\ V1f)\ V3x))\ (ap\ (ap\ (\mathsf{c_2Efinite__map_2EFAPPLY}\ A_27b\ A_27a)\ V2g)\ V3x))))))))) \tag{13}$$

Theorem 1

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0m \in ((A_27a^{A_27a})^{A_27a}).(\forall V1f \in (ty_2Efinite__map_2Efmap\ A_27b\ A_27a).(\forall V2g \in (ty_2Efinite__map_2Efmap\ A_27b\ A_27a). ((ap\ (\mathsf{c_2Efinite__map_2EFDOM}\ A_27b\ A_27a)\ (ap\ (ap\ (ap\ (c_2Efinite__map_2EFMERGE\ A_27a\ A_27b)\ V0m)\ V1f)\ V2g)) = (ap\ (ap\ (\mathsf{c_2Epred__set_2EUNION}\ A_27b)\ (ap\ (\mathsf{c_2Efinite__map_2EFDOM}\ A_27b\ A_27a)\ V1f))\ (ap\ (\mathsf{c_2Efinite__map_2EFDOM}\ A_27b\ A_27a)\ V2g))))))$$