

# thm\_2Efinite\_\_map\_2EFDOM\_\_UNION

(TML8rRmTHq9xd9iibfG4MEsY5p6FRTMMynB)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2E_2T` to be  $(\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^2))) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)$

**Definition 3** We define `c_2Ebool_2E_2IN` to be  $\lambda A. 27a : \iota. (\lambda V0x \in A. 27a. (\lambda V1f \in (2^{A-27a}). (\text{ap } V1f V0x)))$

**Definition 4** We define `c_2Ebool_2E_21` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^{A-27a}))))$

**Definition 5** We define `c_2Ebool_2E_2F` to be  $(\text{ap } (\text{c\_2Ebool\_2E\_21 } 2)) (\lambda V0t \in 2. V0t)$ .

**Definition 6** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 7** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_21 } 2)) (\lambda V2t \in 2. V2t)))$

**Definition 8** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (the (\lambda x. x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 9** We define `c_2Ebool_2ECOND` to be  $\lambda A. 27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A. 27a. (\lambda V2t2 \in A. 27a. (ap V2t2 V1t1))))$

Let `ty_2Eone_2Eone` :  $\iota$  be given. Assume the following.

$$\text{nonempty } ty\_2Eone\_2Eone \tag{1}$$

Let `ty_2Esum_2Esum` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (ty\_2Esum\_2Esum A0 A1) \tag{2}$$

Let `ty_2Efinite__map_2Efmap` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (ty\_2Efinite\_map\_2Efmap A0 A1) \tag{3}$$

Let  $c\_2Efinite\_map\_2Efmap\_REP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_map\_2Efmap\_REP \\ & A\_27a\ A\_27b \in (((ty\_2Esum\_2Esum\ A\_27b\ ty\_2Eone\_2Eone)^{A\_27a})^{(ty\_2Efinite\_map\_2Efmap\ A\_27a\ A\_27b)}) \end{aligned} \quad (4)$$

Let  $c\_2Esum\_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EOUTL \\ & A\_27a\ A\_27b \in (A\_27a^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)}) \end{aligned} \quad (5)$$

**Definition 10** We define  $c\_2Efinite\_map\_2EFAPPLY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (ty\_2Efinite\_map\_2EFUNION$

**Definition 11** We define  $c\_2Ebool\_2E5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V2t \in 2.$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod \\ & A0\ A1) \end{aligned} \quad (6)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ & A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (7)$$

**Definition 12** We define  $c\_2Epair\_2E2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Epair\_2E2C$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ & A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}}) \end{aligned} \quad (8)$$

**Definition 13** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2Epred\_set\_2EUNION$

Let  $c\_2Efinite\_map\_2EFUNION : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_map\_2EFUNION \\ & A\_27a\ A\_27b \in (((ty\_2Efinite\_map\_2Efmap\ A\_27a\ A\_27b)^{(ty\_2Efinite\_map\_2Efmap\ A\_27a\ A\_27b)})^{(ty\_2Efinite\_map\_2Efmap\ A\_27a\ A\_27b)}) \end{aligned} \quad (9)$$

Let  $c\_2Esum\_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EISL \\ & A\_27a\ A\_27b \in (2^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)}) \end{aligned} \quad (10)$$

**Definition 14** We define  $c\_2Efinite\_map\_2EFDOM$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (ty\_2Efinite\_map\_2EFUNION$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A.27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (12)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (13)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow ( \\ & \forall V0f \in (ty.2Efinite\_map.2Efmap A.27a A.27b).(\forall V1g \in \\ & (ty.2Efinite\_map.2Efmap A.27a A.27b).(((ap (c.2Efinite\_map.2EFDOM \\ & A.27a A.27b) (ap (ap (c.2Efinite\_map.2EFUNION A.27a A.27b) V0f) \\ & V1g)) = (ap (ap (c.2Epred\_set.2EUNION A.27a) (ap (c.2Efinite\_map.2EFDOM \\ & A.27a A.27b) V0f)) (ap (c.2Efinite\_map.2EFDOM A.27a A.27b) V1g)))) \wedge \\ & (\forall V2x \in A.27a.((ap (ap (c.2Efinite\_map.2EFAPPLY A.27a \\ & A.27b) (ap (ap (c.2Efinite\_map.2EFUNION A.27a A.27b) V0f) V1g)) \\ & V2x) = (ap (ap (ap (c.2Ebool.2ECOND A.27b) (ap (ap (c.2Ebool.2EIN \\ & A.27a) V2x) (ap (c.2Efinite\_map.2EFDOM A.27a A.27b) V0f)))) (ap \\ & (ap (c.2Efinite\_map.2EFAPPLY A.27a A.27b) V0f) V2x)) (ap (ap ( \\ & c.2Efinite\_map.2EFAPPLY A.27a A.27b) V1g) V2x)))))) \end{aligned} \quad (14)$$

### Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow \forall A.27c. \\ & nonempty A.27c \Rightarrow (\forall V0f \in (ty.2Efinite\_map.2Efmap A.27a \\ & A.27b).(\forall V1g \in (ty.2Efinite\_map.2Efmap A.27a A.27b). \\ & (\forall V2x \in A.27c.((ap (c.2Efinite\_map.2EFDOM A.27a A.27b) \\ & (ap (ap (c.2Efinite\_map.2EFUNION A.27a A.27b) V0f) V1g)) = (ap \\ & (ap (c.2Epred\_set.2EUNION A.27a) (ap (c.2Efinite\_map.2EFDOM \\ & A.27a A.27b) V0f)) (ap (c.2Efinite\_map.2EFDOM A.27a A.27b) V1g)))))) \end{aligned}$$