

thm_2Efinite__map_2EFDOM__f__o (TMY- HDY7pv7cccnFBcUNC7UEtM54np9CW4K6)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2E$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $ty_2Efinite_map_2E fmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efinite_map_2E fmap\ A0\ A1) \tag{3}$$

Let $c_2Efinite_map_2E fmap_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2E fmap_REP\ A_27a\ A_27b \in (((ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a})(ty_2Efinite_map_2E fmap\ A_27a\ A_27b)) \tag{4}$$

Let $c_2Esum_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EOUTL\ A_27a\ A_27b \in (A_27a^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \tag{5}$$

Definition 7 We define $c_2Efinite_map_2EFAPPLY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map$

Definition 8 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A-27a}).(ap\ V1f\ V0x)))$

Definition 9 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (6)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A-27b})^{A-27a}}) \quad (7)$$

Definition 11 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A-27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A-27b}}) \quad (8)$$

Definition 12 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A-27a}).(ap\ (c_2$

Definition 13 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 14 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).(ap\ (c_2Ebool_2E_21\ 2)$

Let $c_2Esum_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EISL\ A_27a\ A_27b \in (2^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \quad (9)$$

Definition 15 We define $c_2Efinite_map_2EFDOM$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map$

Let $c_2Efinite_map_2EFUN_FMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2EFUN_FMAP\ A_27a\ A_27b \in (((ty_2Efinite_map_2E fmap\ A_27a\ A_27b)^{(2^{A-27a})})^{(A_27b^{A-27a})}) \quad (10)$$

Let $c_2Efinite_map_2Ef_o_f : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c.nonempty\ A_27c \Rightarrow c_2Efinite_map_2Ef_o_f\ A_27a\ A_27b\ A_27c \in (((ty_2Efinite_map_2E fmap\ A_27a\ A_27c)^{(ty_2Efinite_map_2E fmap\ A_27a\ A_27b)})^{(ty_2Efinite_map_2E fmap\ A_27a\ A_27b)}) \quad (11)$$

Definition 16 We define $c_2Efinite_map_2Ef_o$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (ty_2Efinite$

Definition 17 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 18 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 19 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Emin_2E_40$

Assume the following.

$$True \tag{12}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{13}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V1x))) \tag{14}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \tag{15}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow \neg (p V0t)))))) \tag{16}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg (p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg (p V0t)))))) \tag{18}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \tag{19}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \Rightarrow (p V1t2)) \wedge ((p V1t2) \Rightarrow (p V0t1)))))) \quad (20)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow \forall A_{.27c}. \\ & nonempty A_{.27c} \Rightarrow (\forall V0f \in (ty_{.2Efinite_map_2E fmap} A_{.27b} \\ & A_{.27c}).(\forall V1g \in (ty_{.2Efinite_map_2E fmap} A_{.27a} A_{.27b}). \\ & (((ap (c_{.2Efinite_map_2EFDOM} A_{.27a} A_{.27c}) (ap (ap (c_{.2Efinite_map_2E f_o_f} \\ & A_{.27a} A_{.27b} A_{.27c}) V0f) V1g)) = (ap (ap (c_{.2Epred_set_2EINTER} A_{.27a}) \\ & (ap (c_{.2Efinite_map_2EFDOM} A_{.27a} A_{.27b}) V1g)) (ap (c_{.2Epred_set_2EGSPEC} \\ & A_{.27a} A_{.27a}) (\lambda V2x \in A_{.27a}.(ap (ap (c_{.2Epair_2E_2C} A_{.27a} 2) \\ & V2x) (ap (ap (c_{.2Ebool_2EIN} A_{.27b}) (ap (ap (c_{.2Efinite_map_2EFAPPLY} \\ & A_{.27a} A_{.27b}) V1g) V2x)) (ap (c_{.2Efinite_map_2EFDOM} A_{.27b} A_{.27c}) \\ & V0f)))))) \wedge (\forall V3x \in A_{.27a}.((p (ap (ap (c_{.2Ebool_2EIN} A_{.27a}) \\ & V3x) (ap (c_{.2Efinite_map_2EFDOM} A_{.27a} A_{.27c}) (ap (ap (c_{.2Efinite_map_2E f_o_f} \\ & A_{.27a} A_{.27b} A_{.27c}) V0f) V1g)))))) \Rightarrow ((ap (ap (c_{.2Efinite_map_2EFAPPLY} \\ & A_{.27a} A_{.27c}) (ap (ap (c_{.2Efinite_map_2E f_o_f} A_{.27a} A_{.27b} A_{.27c}) \\ & V0f) V1g)) V3x) = (ap (ap (c_{.2Efinite_map_2EFAPPLY} A_{.27b} A_{.27c}) \\ & V0f) (ap (ap (c_{.2Efinite_map_2EFAPPLY} A_{.27a} A_{.27b}) V1g) V3x))))))))) \quad (22) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\\ & \forall V0f \in (A_{.27b}^{A_{.27a}}).(\forall V1P \in (2^{A_{.27a}}).((p (ap (c_{.2Epred_set_2EFINITE} \\ & A_{.27a}) V1P)) \Rightarrow (((ap (c_{.2Efinite_map_2EFDOM} A_{.27a} A_{.27b}) (ap (\\ & ap (c_{.2Efinite_map_2EFUN_FMAP} A_{.27a} A_{.27b}) V0f) V1P)) = V1P) \wedge \\ & (\forall V2x \in A_{.27a}.((p (ap (ap (c_{.2Ebool_2EIN} A_{.27a}) V2x) V1P)) \Rightarrow \\ & ((ap (ap (c_{.2Efinite_map_2EFAPPLY} A_{.27a} A_{.27b}) (ap (ap (c_{.2Efinite_map_2EFUN_FMAP} \\ & A_{.27a} A_{.27b}) V0f) V1P)) V2x) = (ap V0f V2x))))))))) \quad (23) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\\ & \forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.(\forall V2a \in A_{.27a}.(\forall V3b \in \\ & A_{.27b}.(((ap (ap (c_{.2Epair_2E_2C} A_{.27a} A_{.27b}) V0x) V1y) = (ap (ap \\ & (c_{.2Epair_2E_2C} A_{.27a} A_{.27b}) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \quad (24) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN \\ & A_27a) V2x) V0s)) \Leftrightarrow (p (ap (ap (c_2Ebool_2EIN\ A_27a) V2x) V1t))))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}). (\forall V1v \in \\ & A_27a. ((p (ap (ap (c_2Ebool_2EIN\ A_27a) V1v) (ap (c_2Epred_set_2EGSPEC \\ & A_27a\ A_27b) V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap (ap (c_2Epair_2E_2C \\ & A_27a\ 2) V1v) c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). (\forall V2x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN\ A_27a) \\ & V2x) (ap (ap (c_2Epred_set_2EINTER\ A_27a) V0s) V1t))) \Leftrightarrow ((p (ap \quad (27) \\ & (ap (c_2Ebool_2EIN\ A_27a) V2x) V0s)) \wedge (p (ap (ap (c_2Ebool_2EIN \\ & A_27a) V2x) V1t)))))) \end{aligned}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow (\forall V0f \in (ty_2Efinite_map_2E fmap\ A_27b \\ & A_27c). (\forall V1g \in (A_27b^{A_27a}). ((p (ap (c_2Epred_set_2EFINITE \\ & A_27a) (ap (c_2Epred_set_2EGSPEC\ A_27a\ A_27a) (\lambda V2x \in A_27a. \\ & (ap (ap (c_2Epair_2E_2C\ A_27a\ 2) V2x) (ap (ap (c_2Ebool_2EIN\ A_27b) \\ & (ap\ V1g\ V2x)) (ap (c_2Efinite_map_2EFDOM\ A_27b\ A_27c) V0f)))))) \Rightarrow \\ & ((ap (c_2Efinite_map_2EFDOM\ A_27a\ A_27c) (ap (ap (c_2Efinite_map_2Ef_o \\ & A_27a\ A_27b\ A_27c) V0f) V1g)) = (ap (c_2Epred_set_2EGSPEC\ A_27a \\ & A_27a) (\lambda V3x \in A_27a. (ap (ap (c_2Epair_2E_2C\ A_27a\ 2) V3x) (\\ & ap (ap (c_2Ebool_2EIN\ A_27b) (ap\ V1g\ V3x)) (ap (c_2Efinite_map_2EFDOM \\ & A_27b\ A_27c) V0f)))))) \end{aligned}$$