

thm_2Efinite_map_2EFEVERY_DRESTRICT_COMPL
 (TMaXLy-
 DAx2FRKp24S6v1gcWGM19yLXvQd67)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p (ap P x)))$ of type $\iota \Rightarrow \iota$.

Definition 4 We define $c_2Ebool_2E_3F$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A))))$

Let $ty_2Efinite_map_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efinite_map_2Efmap A0 A1) \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{2}$$

Let $c_2Efinite_map_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Efinite_map_2EFUPDATE A.27a A.27b \in (((ty_2Efinite_map_2Efmap A.27a A.27b)^{(ty_2Epair_2Eprod A.27a A.27b)})^{(ty_2Efinite_map_2EFUPDATE A.27a A.27b)}) \tag{3}$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty ty_2Eone_2Eone \tag{4}$$

Definition 5 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone.V0x))$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 7 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p P \Rightarrow q Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Esum_2Esum A0 A1) \quad (5)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EABS_sum A_27a A_27b \in ((ty_2Esum_2Esum A_27a A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (6)$$

Definition 11 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_2Esum_2EABS$

Let $c_2Efinite_map_2Efmap_ABS : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2Efmap_ABS A_27a A_27b \in ((ty_2Efinite_map_2Efmap A_27a A_27b)^{(ty_2Esum_2Esum A_27b ty_2Eone_2Eone)^{A_27a}}) \quad (7)$$

Definition 12 We define $c_2Efinite_map_2EFEMPTY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (ap (c_2Efinite_map_2E$

Definition 13 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Let $c_2Efinite_map_2EDRESTRICT : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2EDRESTRICT A_27a A_27b \in ((ty_2Efinite_map_2Efmap A_27a A_27b)^{(2^{A_27a})})^{(ty_2Efinite_map_2Efmap A_27a A_27b)} \quad (8)$$

Let $c_2Efinite_map_2Efmap_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2Efmap_REP A_27a A_27b \in ((ty_2Esum_2Esum A_27b ty_2Eone_2Eone)^{A_27a})^{(ty_2Efinite_map_2Efmap A_27a A_27b)} \quad (9)$$

Let $c_2Esum_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EOUTL A_27a A_27b \in (A_27a)^{(ty_2Esum_2Esum A_27a A_27b)} \quad (10)$$

Definition 14 We define $c_2Efinite_map_2EFAPPLY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map_2EFAPPLY)$.
Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (11)$$

Definition 15 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2E_2C\ x\ y))$.
Let $c_2Esum_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EISL \\ A_27a\ A_27b \in (2^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \end{aligned} \quad (12)$$

Definition 16 We define $c_2Efinite_map_2EFDOM$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map_2EFDOM)$.

Definition 17 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$.

Definition 18 We define $c_2Efinite_map_2EFEVERY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$.

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (13)$$

Definition 19 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EINTER\ s\ t))$.

Definition 20 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (V0t1\ V1t2))))$.

Definition 21 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap\ (c_2Epred_set_2EINSERT\ x\ s))$.

Definition 22 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E_2ET)$.

Definition 23 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EDIFF\ s\ t))$.

Definition 24 We define $c_2Epred_set_2Ecompl$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Epred_set_2Ecompl\ P)))$.

Assume the following.

$$True \quad (14)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.((p \ V0t) \vee (\neg(p \ V0t)))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((p \ V0t) \Rightarrow \text{False}) \Rightarrow (\neg(p \ V0t)))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p \ V0t)) \Rightarrow ((p \ V0t) \Rightarrow \text{False}))) \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((\text{True} \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge \text{True}) \Leftrightarrow \\ & (p \ V0t)) \wedge (((\text{False} \wedge (p \ V0t)) \Leftrightarrow \text{False}) \wedge (((p \ V0t) \wedge \text{False}) \Leftrightarrow \text{False}) \wedge \\ & (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((\text{True} \vee (p \ V0t)) \Leftrightarrow \text{True}) \wedge (((p \ V0t) \vee \text{True}) \Leftrightarrow \text{True}) \wedge \\ & (((\text{False} \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \text{False}) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\ & (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((\text{True} \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow \text{True}) \Leftrightarrow \\ & \text{True}) \wedge (((\text{False} \Rightarrow (p \ V0t)) \Leftrightarrow \text{True}) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow \text{True}) \wedge ((\\ & (p \ V0t) \Rightarrow \text{False}) \Leftrightarrow (\neg(p \ V0t)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge (((\neg \text{True}) \Leftrightarrow \text{False}) \wedge \\ & ((\neg \text{False}) \Leftrightarrow \text{True})))) \end{aligned} \quad (23)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.((V0x = V0x) \Leftrightarrow \text{True})) \quad (24)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in A_{.27a}.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((\text{True} \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow \text{True}) \Leftrightarrow \\ & (p \ V0t)) \wedge (((\text{False} \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow \text{False}) \Leftrightarrow (\neg(\\ & p \ V0t)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0t1 \in A.27a. (\forall V1t2 \in \\ A.27a. (((ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ A.27a)\ c.2Ebool.2ET)\ V0t1) \\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ A.27a)\ c.2Ebool.2EF) \\ V0t1)\ V1t2) = V1t2)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). ((\neg(\forall V1x \in \\ A.27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A.27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (28)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in \\ (2^{A.27a}). ((\forall V2x \in A.27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow \\ ((\forall V3x \in A.27a. (p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A.27a. (p\ (\\ ap\ V1Q\ V4x))))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in \\ (2^{A.27a}). ((\exists V2x \in A.27a. ((p\ (ap\ V0P\ V2x)) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow \\ ((\exists V3x \in A.27a. (p\ (ap\ V0P\ V3x))) \vee (\exists V4x \in A.27a. (p\ (\\ ap\ V1Q\ V4x))))))) \end{aligned} \quad (30)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ 2^{A.27a}). ((p\ V0P) \vee (\exists V2x \in A.27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\exists V3x \in \\ A.27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V3x)))))) \quad (31)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in \\ 2. ((\exists V2x \in A.27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ V1Q))) \Leftrightarrow ((\exists V3x \in \\ A.27a. (p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \quad (32)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ 2^{A.27a}). ((\exists V2x \in A.27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p \\ V0P) \wedge (\exists V3x \in A.27a. (p\ (ap\ V1Q\ V3x)))))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (\\ (p\ V1B) \vee (p\ V2C)))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B))))))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V1B) \vee (p V2C)) \wedge (p V0A)) \Leftrightarrow (((p V1B) \wedge (p V0A)) \vee ((p V2C) \wedge (p V0A)))))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))) \quad (38)$$

Assume the following.

$$(\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2R \in 2.(((p V0P) \vee (p V1Q)) \Rightarrow (p V2R)) \Leftrightarrow (((p V0P) \Rightarrow (p V2R)) \wedge ((p V1Q) \Rightarrow (p V2R)))))) \quad (39)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (40)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\\ \forall V0f \in (A.27b^{A.27a}).(\forall V1b \in 2.(\forall V2x \in A.27a. \\ (\forall V3y \in A.27a.((ap V0f (ap (ap (ap (c.2Ebool.2ECOND A.27a) \\ V1b) V2x) V3y)) = (ap (ap (ap (c.2Ebool.2ECOND A.27b) V1b) (ap V0f \\ V2x)) (ap V0f V3y)))))) \end{aligned} \quad (41)$$

Assume the following.

$$(\forall V0b \in 2.(\forall V1t1 \in 2.(\forall V2t2 \in 2.((p (ap (ap (ap (c.2Ebool.2ECOND 2) V0b) V1t1) V2t2)) \Leftrightarrow (((\neg(p V0b)) \vee (p V1t1)) \wedge ((p V0b) \vee (p V2t2)))))) \quad (42)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in 2.(((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \quad (43)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& (\forall V2x \in A.27a. (\forall V3x.27 \in A.27a. (\forall V4y \in A.27a. \\
& (\forall V5y.27 \in A.27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x.27)) \wedge \\
& ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y.27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ A.27a) \\
& V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ A.27a)\ V1Q)\ V3x.27) \\
& V5y.27)))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (2^{A.27a}). (\forall V1v \in \\
& A.27a. ((\forall V2x \in A.27a. ((V2x = V1v) \Rightarrow (p\ (ap\ V0f\ V2x)))) \Leftrightarrow (p\ (\\
& ap\ V0f\ V1v))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0f \in (ty.2Efinite_map.2Efmap\ A.27a\ A.27b). (\forall V1a \in \\
& A.27a. (\forall V2b \in A.27b. ((ap\ (c.2Efinite_map.2EFDOM\ A.27a \\
& A.27b)\ (ap\ (ap\ (c.2Efinite_map.2EFUPDATE\ A.27a\ A.27b)\ V0f)\ (ap \\
& (ap\ (c.2Epair.2E.2C\ A.27a\ A.27b)\ V1a)\ V2b))) = (ap\ (ap\ (c.2Epred_set.2EINSERT \\
& A.27a)\ V1a)\ (ap\ (c.2Efinite_map.2EFDOM\ A.27a\ A.27b)\ V0f))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0f \in (ty.2Efinite_map.2Efmap\ A.27a\ A.27b). (\forall V1a \in \\
& A.27a. (\forall V2b \in A.27b. (\forall V3x \in A.27a. ((ap\ (ap\ (c.2Efinite_map.2EFAPPLY \\
& A.27a\ A.27b)\ (ap\ (ap\ (c.2Efinite_map.2EFUPDATE\ A.27a\ A.27b)\ V0f)\ \\
& (ap\ (ap\ (c.2Epair.2E.2C\ A.27a\ A.27b)\ V1a)\ V2b)))\ V3x) = (ap\ (ap\ (ap \\
& (c.2Ebool.2ECOND\ A.27b)\ (ap\ (ap\ (c.2Emin.2E.3D\ A.27a)\ V3x)\ V1a)) \\
& V2b)\ (ap\ (ap\ (c.2Efinite_map.2EFAPPLY\ A.27a\ A.27b)\ V0f)\ V3x))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0f \in (ty.2Efinite_map.2Efmap\ A.27a\ A.27b). (\forall V1r \in \\
& (2^{A.27a}). (((ap\ (c.2Efinite_map.2EFDOM\ A.27a\ A.27b)\ (ap\ (ap \\
& (c.2Efinite_map.2EDRESTRICT\ A.27a\ A.27b)\ V0f)\ V1r)) = (ap\ (ap \\
& (c.2Epred_set.2EINTER\ A.27a)\ (ap\ (c.2Efinite_map.2EFDOM\ A.27a \\
& A.27b)\ V0f))\ V1r)) \wedge (\forall V2x \in A.27a. ((ap\ (ap\ (c.2Efinite_map.2EFAPPLY \\
& A.27a\ A.27b)\ (ap\ (ap\ (c.2Efinite_map.2EDRESTRICT\ A.27a\ A.27b) \\
& V0f)\ V1r))\ V2x) = (ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ A.27b)\ (ap\ (ap\ (c.2Ebool.2EIN \\
& A.27a)\ V2x)\ (ap\ (ap\ (c.2Epred_set.2EINTER\ A.27a)\ (ap\ (c.2Efinite_map.2EFDOM \\
& A.27a\ A.27b)\ V0f))\ V1r)))\ (ap\ (ap\ (c.2Efinite_map.2EFAPPLY\ A.27a \\
& A.27b)\ V0f)\ V2x))\ (ap\ (ap\ (c.2Efinite_map.2EFAPPLY\ A.27a\ A.27b) \\
& (c.2Efinite_map.2EFEMPTY\ A.27a\ A.27b))\ V2x))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow (\forall V0f \in (ty_2Efinite_map_2Efmap\ A_27a \\
& A_27b).(\forall V1r \in (2^{A_27a}).(\forall V2x \in A_27c.((ap\ (c_2Efinite_map_2EFDOM \\
& A_27a\ A_27b)\ (ap\ (ap\ (c_2Efinite_map_2EDRESTRICT\ A_27a\ A_27b) \\
& V0f)\ V1r))) = (ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a)\ (ap\ (c_2Efinite_map_2EFDOM \\
& A_27a\ A_27b)\ V0f))\ V1r))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1t \in \\
& (2^{A_27a}).(\forall V2x \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\
& V2x)\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a)\ V0s)\ V1t)))) \Leftrightarrow ((p\ (ap \\
& (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& A_27a)\ V2x)\ V1t))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\
& A_27a.(\forall V2s \in (2^{A_27a}).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\
& V0x)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V1y)\ V2s)))) \Leftrightarrow ((V0x = \\
& V1y) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ V2s))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1s \in \\
& (2^{A_27a}).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (ap\ (c_2Epred_set_2Ecompl \\
& A_27a)\ V1s)))) \Leftrightarrow (\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ V1s))))))
\end{aligned} \tag{52}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{53}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{56}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Leftrightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee ((\neg(\\
& p \ V2r)) \vee (\neg(p \ V1q)))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V0p)))) \wedge ((p \ V2r) \vee \\
& ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ((\\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{62}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{63}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{64}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V0p)))) \tag{65}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{66}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \tag{67}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0P \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}). (\forall V1f \in \\ & \quad (ty_2Efinite_map_2Efmap\ A_27a\ A_27b). (\forall V2k \in A_27a. (\\ & \quad \forall V3v \in A_27b. (\forall V4s \in (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Efinite_map_2EFEVERY \\ & \quad A_27a\ A_27b)\ V0P)\ (ap\ (ap\ (c_2Efinite_map_2EDRESTRICT\ A_27a\ A_27b) \\ & \quad (ap\ (ap\ (c_2Efinite_map_2EFUPDATE\ A_27a\ A_27b)\ V1f)\ (ap\ (ap\ (c_2Epair_2E_2C \\ & \quad A_27a\ A_27b)\ V2k)\ V3v)))\ (ap\ (c_2Epred_set_2ECOMPL\ A_27a)\ V4s)))) \Leftrightarrow \\ & \quad (((\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2k)\ V4s))) \Rightarrow (p\ (ap\ V0P\ (ap\ (\\ & \quad ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2k)\ V3v)))) \wedge (p\ (ap\ (ap\ (c_2Efinite_map_2EFEVERY \\ & \quad A_27a\ A_27b)\ V0P)\ (ap\ (ap\ (c_2Efinite_map_2EDRESTRICT\ A_27a\ A_27b) \\ & \quad V1f)\ (ap\ (c_2Epred_set_2ECOMPL\ A_27a)\ (ap\ (ap\ (c_2Epred_set_2EINSERT \\ & \quad A_27a)\ V2k)\ V4s))))))))))))) \end{aligned}$$