

thm_2Efinite_map_2EFEVERY_UPDATE_LIST
(TMG-
bCw7byQP1Yk7M8T2ZfMgNWo6hizwdNbc)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Ebool_2EBOUNDED$ to be $(\lambda V0v \in 2.c_2Ebool_2ET)$.

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p (ap P x))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_2Emin_2E_40 (2^{A_27a}))$

Let $ty_2Efinite_map_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efinite_map_2Efmap A0 A1) \quad (1)$$

Let $c_2Efinite_map_2EDRESTRICT : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2EDRESTRICT A_27a A_27b \in (((ty_2Efinite_map_2Efmap A_27a A_27b)^{(2^{A_27a}}))^{(ty_2Efinite_map_2Efmap A_27a A_27b)}) \quad (2)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (3)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (4)$$

Let $c_2Efinite_map_2Efmap_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2Efmap_REP\ A_27a\ A_27b \in (((ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a})^{(ty_2Efinite_map_2Efmap\ A_27a\ A_27b)}) \quad (5)$$

Let $c_2Esum_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EOUTL\ A_27a\ A_27b \in (A_27a^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \quad (6)$$

Definition 10 We define $c_2Efinite_map_2EFAPPLY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map_2EFmap\ A_27a\ A_27b).$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (7)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (8)$$

Definition 11 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2Eprod\ A_27a\ A_27b)\ V0x\ V1y)$

Let $c_2Esum_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EISL\ A_27a\ A_27b \in (2^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \quad (9)$$

Definition 12 We define $c_2Efinite_map_2EFDOM$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map_2EFmap\ A_27a\ A_27b).$

Definition 13 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 14 We define $c_2Efinite_map_2EFEVERY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$

Let $c_2Efinite_map_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2EFUPDATE\ A_27a\ A_27b \in (((ty_2Efinite_map_2Efmap\ A_27a\ A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})^{(ty_2Efinite_map_2EFmap\ A_27a\ A_27b)}) \quad (10)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (11)$$

Let $c_2Elist_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EFOLDL \\ A_27a\ A_27b \in (((A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{A_27b})^{((A_27b)^{A_27a})^{A_27b}} \end{aligned} \quad (12)$$

Definition 15 We define $c_2Efinite_map_2EFUPDATE_LIST$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (ap\ (c_2Elist_2Elist\ A_27a)\ (c_2Elist_2Elist\ A_27b))$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EMAP \\ A_27a\ A_27b \in (((ty_2Elist_2Elist\ A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{A_27b})^{A_27a} \end{aligned} \quad (13)$$

Let $c_2Elist_2EEVERY : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EEVERY\ A_27a \in ((2^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (14)$$

Let $c_2Elist_2ELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELIST_TO_SET\ A_27a \in ((2^{A_27a})^{(ty_2Elist_2Elist\ A_27a)}) \quad (15)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (16)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (17)$$

Let $c_2Elist_2EALL_DISTINCT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EALL_DISTINCT\ A_27a \in (2^{(ty_2Elist_2Elist\ A_27a)}) \quad (18)$$

Definition 16 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a)\ (c_2Emin_2E_40\ A_27b))))$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \end{aligned} \quad (19)$$

Definition 17 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (c_2Ebool_2E_21\ 2))))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{((\neg 2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \end{aligned} \quad (20)$$

Definition 18 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap\ (c_2Epred_set_2EINSERT\ V0x\ s))$

Definition 19 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E\ V0t))$

Definition 20 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E_7E\ V0x)$

Definition 21 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E_7E\ V0x)$

Definition 22 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EINTER\ V0s\ V1t))$

Definition 23 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EUNION\ V0s\ V1t))$

Definition 24 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EDIFF\ V0s\ V1t))$

Definition 25 We define $c_2Epred_set_2Ecompl$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Epred_set_2Ecompl\ V0P)))$

Assume the following.

$$True \quad (21)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (23)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} ((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (29)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2) \Rightarrow (p\ V2t3)) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (30)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Leftrightarrow (p\ V1t2)) \Leftrightarrow (((p\ V0t1) \Rightarrow (p\ V1t2)) \wedge ((p\ V1t2) \Rightarrow (p\ V0t1)))))) \quad (31)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27)\ V5y_27)))))) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V2t1)\ V3t2) = V3t2)))) \quad (34)$$

Assume the following.

$$(\forall V0v \in 2. ((p\ (ap\ c_2Ebool_2EBOUNDED\ V0v)) \Leftrightarrow True)) \quad (35)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (ty_2Efinite_map_2Efmap\ A_27a\ A_27b).(\forall V1r \in \\
& (2^{A_27a}).(\forall V2x \in A_27a.(\forall V3y \in A_27b.((ap\ (ap\ (c_2Efinite_map_2EDRESTRICT \\
& \quad A_27a\ A_27b)\ (ap\ (ap\ (c_2Efinite_map_2EFUPDATE\ A_27a\ A_27b)\ V0f) \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2x)\ V3y)))\ V1r) = (ap\ (ap\ (ap \\
& \quad (c_2Ebool_2ECOND\ (ty_2Efinite_map_2Efmap\ A_27a\ A_27b))\ (ap \\
& (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1r))\ (ap\ (ap\ (c_2Efinite_map_2EFUPDATE \\
& \quad A_27a\ A_27b)\ (ap\ (ap\ (c_2Efinite_map_2EDRESTRICT\ A_27a\ A_27b) \\
& \quad V0f)\ V1r))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2x)\ V3y)))\ (ap\ (\\
& \quad ap\ (c_2Efinite_map_2EDRESTRICT\ A_27a\ A_27b)\ V0f)\ V1r)))))) \\
& \hspace{15em} (36)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (ty_2Efinite_map_2Efmap\ A_27a\ A_27b).(\forall V1P \in \\
& (2^{A_27a}).(\forall V2Q \in (2^{A_27a}).((ap\ (ap\ (c_2Efinite_map_2EDRESTRICT \\
& \quad A_27a\ A_27b)\ (ap\ (ap\ (c_2Efinite_map_2EDRESTRICT\ A_27a\ A_27b) \\
& \quad V0f)\ V1P))\ V2Q) = (ap\ (ap\ (c_2Efinite_map_2EDRESTRICT\ A_27a\ A_27b) \\
& \quad V0f)\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a)\ V1P)\ V2Q)))))) \\
& \hspace{15em} (37)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (ty_2Efinite_map_2Efmap\ A_27a\ A_27b).((ap\ (ap\ (\\
& \quad c_2Efinite_map_2EDRESTRICT\ A_27a\ A_27b)\ V0f)\ (c_2Epred_set_2EUNIV \\
& \quad A_27a)) = V0f)) \\
& \hspace{15em} (38)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0P \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}).(\forall V1f \in \\
& (ty_2Efinite_map_2Efmap\ A_27a\ A_27b).(\forall V2x \in A_27a.(\\
& \quad \forall V3y \in A_27b.((p\ (ap\ (ap\ (c_2Efinite_map_2EFEVERY\ A_27a \\
& \quad A_27b)\ V0P)\ (ap\ (ap\ (c_2Efinite_map_2EFUPDATE\ A_27a\ A_27b)\ V1f) \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2x)\ V3y)))) \Leftrightarrow ((p\ (ap\ V0P)\ (ap \\
& (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2x)\ V3y))) \wedge (p\ (ap\ (ap\ (c_2Efinite_map_2EFEVERY \\
& \quad A_27a\ A_27b)\ V0P)\ (ap\ (ap\ (c_2Efinite_map_2EDRESTRICT\ A_27a\ A_27b) \\
& \quad V1f)\ (ap\ (c_2Epred_set_2ECOMPL\ A_27a)\ (ap\ (ap\ (c_2Epred_set_2EINSERT \\
& \quad A_27a)\ V2x)\ (c_2Epred_set_2EEMPTY\ A_27a)))))))))) \\
& \hspace{15em} (39)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (ty_2Efinite_map_2Efmap\ A_27a\ A_27b).((ap\ (ap \\
& \quad (c_2Efinite_map_2EFUPDATE_LIST\ A_27a\ A_27b)\ V0f)\ (c_2Elist_2ENIL \\
& \quad (ty_2Epair_2Eprod\ A_27a\ A_27b)))) = V0f) \wedge (\forall V1h \in (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27b).(\forall V2t \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27b)).((ap\ (ap\ (c_2Efinite_map_2EFUPDATE_LIST\ A_27a \\
& \quad A_27b)\ V0f)\ (ap\ (ap\ (c_2Elist_2ECONS\ (ty_2Epair_2Eprod\ A_27a\ A_27b)) \\
& \quad V1h)\ V2t))) = (ap\ (ap\ (c_2Efinite_map_2EFUPDATE_LIST\ A_27a\ A_27b) \\
& \quad (ap\ (ap\ (c_2Efinite_map_2EFUPDATE\ A_27a\ A_27b)\ V0f)\ V1h))\ V2t))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad (\forall V0f \in (A_27b^{A_27a}).((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b) \\
& \quad V0f)\ (c_2Elist_2ENIL\ A_27a)) = (c_2Elist_2ENIL\ A_27b))) \wedge (\forall V1f \in \\
& \quad (A_27b^{A_27a}).(\forall V2h \in A_27a.(\forall V3t \in (ty_2Elist_2Elist \\
& \quad A_27a).((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad A_27a)\ V2h)\ V3t))) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27b)\ (ap\ V1f\ V2h)) \\
& \quad (ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ V3t))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0h \in A_27b.(\forall V1t \in (ty_2Elist_2Elist\ A_27b).((\\
& \quad (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ (c_2Elist_2ENIL\ A_27a)) = \\
& \quad (c_2Epred_set_2EEMPTY\ A_27a)) \wedge ((ap\ (c_2Elist_2ELIST_TO_SET \\
& \quad A_27b)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27b)\ V0h)\ V1t)) = (ap\ (ap\ (c_2Epred_set_2EINSERT \\
& \quad A_27b)\ V0h)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27b)\ V1t))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0P \in (2^{A_27a}).((p\ (ap \\
& \quad (ap\ (c_2Elist_2EVERY\ A_27a)\ V0P)\ (c_2Elist_2ENIL\ A_27a))) \Leftrightarrow True)) \wedge \\
& \quad (\forall V1P \in (2^{A_27a}).(\forall V2h \in A_27a.(\forall V3t \in (ty_2Elist_2Elist \\
& \quad A_27a).((p\ (ap\ (ap\ (c_2Elist_2EVERY\ A_27a)\ V1P)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad A_27a)\ V2h)\ V3t))) \Leftrightarrow ((p\ (ap\ V1P\ V2h)) \wedge (p\ (ap\ (ap\ (c_2Elist_2EVERY \\
& \quad A_27a)\ V1P)\ V3t))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\
& \quad (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\
& \quad A_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\
& \quad c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\
& \quad A_27a).(p\ (ap\ V0P\ V3l))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & ((p\ (ap\ (c.2Elist.2EALL_DISTINCT \\ & A.27a)\ (c.2Elist.2ENIL\ A.27a))) \Leftrightarrow True) \wedge (\forall V0h \in A.27a. (\\ \forall V1t \in (ty.2Elist.2Elist\ A.27a). & ((p\ (ap\ (c.2Elist.2EALL_DISTINCT \\ & A.27a)\ (ap\ (ap\ (c.2Elist.2ECONS\ A.27a)\ V0h)\ V1t))) \Leftrightarrow ((\neg(p\ (ap\ (ap \\ & (c.2Ebool.2EIN\ A.27a)\ V0h)\ (ap\ (c.2Elist.2ELIST_TO_SET\ A.27a) \\ & V1t)))) \wedge (p\ (ap\ (c.2Elist.2EALL_DISTINCT\ A.27a)\ V1t)))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & \forall A.27b.nonempty\ A.27b \Rightarrow (\\ \forall V0x \in (ty.2Epair.2Eprod\ A.27a\ A.27b). & (\exists V1q \in A.27a. \\ (\exists V2r \in A.27b. & (V0x = (ap\ (ap\ (c.2Epair.2E.2C\ A.27a\ A.27b) \\ & V1q)\ V2r)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & \forall A.27b.nonempty\ A.27b \Rightarrow (\\ \forall V0x \in A.27a. & (\forall V1y \in A.27b. ((ap\ (c.2Epair.2E.FST\ A.27a \\ & A.27b)\ (ap\ (ap\ (c.2Epair.2E.2C\ A.27a\ A.27b)\ V0x)\ V1y)) = V0x))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0s \in (2^{A.27a}). (\forall V1t \in \\ (2^{A.27a}). & ((ap\ (ap\ (c.2Epred_set.2EUNION\ A.27a)\ V0s)\ V1t) = (\\ & ap\ (ap\ (c.2Epred_set.2EUNION\ A.27a)\ V1t)\ V0s)))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0s \in (2^{A.27a}). (\forall V1x \in \\ A.27a. & ((ap\ (ap\ (c.2Epred_set.2EINSERT\ A.27a)\ V1x)\ V0s) = (ap\ (\\ & ap\ (c.2Epred_set.2EUNION\ A.27a)\ (ap\ (ap\ (c.2Epred_set.2EINSERT \\ & A.27a)\ V1x)\ (c.2Epred_set.2EEMPTY\ A.27a)))\ V0s)))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0x \in A.27a. (\forall V1s \in \\ (2^{A.27a}). & ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V0x)\ (ap\ (c.2Epred_set.2Ecompl \\ & A.27a)\ V1s))) \Leftrightarrow (\neg(p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V0x)\ V1s)))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & ((ap\ (c.2Epred_set.2Ecompl\ A.27a) \\ (c.2Epred_set.2EEMPTY\ A.27a)) & = (c.2Epred_set.2Euniv\ A.27a)) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0s \in (2^{A.27a}). (\forall V1t \in \\ (2^{A.27a}). & ((ap\ (c.2Epred_set.2Ecompl\ A.27a)\ (ap\ (ap\ (c.2Epred_set.2Eunion \\ & A.27a)\ V0s)\ V1t)) = (ap\ (ap\ (c.2Epred_set.2Einter\ A.27a)\ (ap\ (c.2Epred_set.2Ecompl \\ & A.27a)\ V0s))\ (ap\ (c.2Epred_set.2Ecompl\ A.27a)\ V1t)))))) \end{aligned} \quad (52)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0kvl \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b)). \\ & (\forall V1P \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})). (\forall V2fm \in \\ & (ty_2Efinite_map_2E fmap\ A_27a\ A_27b)). ((p\ (ap\ (c_2Elist_2EALL_DISTINCT \\ & A_27a)\ (ap\ (ap\ (c_2Elist_2EMAP\ (ty_2Epair_2Eprod\ A_27a\ A_27b) \\ & A_27a)\ (c_2Epair_2EFST\ A_27a\ A_27b))\ V0kvl))) \Rightarrow ((p\ (ap\ (ap\ (c_2Efinite_map_2EFEVERY \\ & A_27a\ A_27b)\ V1P)\ (ap\ (ap\ (c_2Efinite_map_2EFUPDATE_LIST\ A_27a \\ & A_27b)\ V2fm)\ V0kvl))) \Leftrightarrow ((p\ (ap\ (ap\ (c_2Elist_2EEVERY\ (ty_2Epair_2Eprod \\ & A_27a\ A_27b))\ V1P)\ V0kvl)) \wedge (p\ (ap\ (ap\ (c_2Efinite_map_2EFEVERY \\ & A_27a\ A_27b)\ V1P)\ (ap\ (ap\ (c_2Efinite_map_2EDRESTRICT\ A_27a\ A_27b) \\ & V2fm)\ (ap\ (c_2Epred_set_2E COMPL\ A_27a)\ (ap\ (c_2Elist_2ELIST_TO_SET \\ & A_27a)\ (ap\ (ap\ (c_2Elist_2EMAP\ (ty_2Epair_2Eprod\ A_27a\ A_27b) \\ & A_27a)\ (c_2Epair_2EFST\ A_27a\ A_27b))\ V0kvl))))))))))))) \end{aligned}$$