thm_2Efinite__map_2EFINITE__PRED__11 (TMYtJSyC-QVZPuKUEAvo4mDGuBKD1MH4Lwxh)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 4 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Let $ty\_2Efinite\_\_map\_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efinite\_\_map\_2Efmap\ A0\ A1) \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $c\_2Efinite\_\_map\_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_\_map\_2EFUPDAT\ A\_27a\ A\_27b \in (((ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})^{(ty\_2Efinite\_\_map} \tag{3}$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{4}$$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{5}$$

Let $c\_2Efinite\_\_map\_2Efmap\_\_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_\_map\_2Efmap\_\_REP\ A\_27a\ A\_27b \in (((ty\_2Esum\_2Esum\ A\_27b\ ty\_2Eone\_2Eone)^{A\_27a})^{(ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b)}) \quad (6)$$

Let $c\_2Esum\_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EISL\ A\_27a\ A\_27b \in (2^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)}) \quad (7)$$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27a}$

Definition 6 *We define* c_2Efinite__map_2EFDOM *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (ty\_2Efinite\_\_map\_2$

Definition 7 *We define* c_2Eone_2Eone *to be* $(ap\ ($c_2Emin_2E_40 $ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2E$

Definition 8 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Definition 9 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 10 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 11 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \quad (8)$$

Definition 12 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $c\_2Efinite\_\_map\_2Efmap\_\_ABS : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_\_map\_2Efmap\_\_ABS\ A\_27a\ A\_27b \in ((ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b)^{((ty\_2Esum\_2Esum\ A\_27b\ ty\_2Eone\_2Eone)^{A\_27a})}) \quad (9)$$

Definition 13 *We define* c_2Efinite__map_2EFEMPTY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(ap\ (c\_2Efinite\_\_map\_2E$

Definition 14 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 15 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (10)$$

Definition 16 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{11}$$

Definition 17 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 18 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 19 *We define* c_2Epred__set_2EFINITE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap$ (c_2Ebool_2E_21 (2

Definition 20 *We define* c_2Epred__set_2ESING *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap$ (c_2Ebool_2E_3F A_2

Definition 21 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c$

Assume the following.

$$True \tag{12}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{13}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{14}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \lor (\neg (p\ V0t)))) \tag{15}$$

Assume the following.

$$(\forall V0t \in 2.(((True \land (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \land True) \Leftrightarrow (p\ V0t)) \land (((False \land (p\ V0t)) \Leftrightarrow False) \land ((((p\ V0t) \land False) \Leftrightarrow False) \land (((p\ V0t) \land (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \land (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \land ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \land (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{18}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg (\exists V1x \in A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg (p\ (ap\ V0P\ V2x)))))) \tag{20}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{21}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((ap\ (\mathsf{c\_2Efinite\_\_map\_2EFDOM}\ A\_27a\ A\_27b)\ (\mathsf{c\_2Efinite\_\_map\_2EFEMPTY}\ A\_27a\ A\_27b)) = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b).(\forall V1a \in A\_27a.(\forall V2b \in A\_27b.((ap\ (\mathsf{c\_2Efinite\_\_map\_2EFDOM}\ A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Efinite\_\_map\_2EFUPDATE\ A\_27a\ A\_27b)\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V1a)\ V2b))) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V1a)\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFDOM}\ A\_27a\ A\_27b)\ V0f)))))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0P \in (2^{(ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b)}).(((p\ (ap\ V0P\ (\mathsf{c\_2Efinite\_\_map\_2EFEMPTY}\ A\_27a\ A\_27b))) \wedge (\forall V1f \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b).((p\ (ap\ V0P\ V1f)) \Rightarrow (\forall V2x \in A\_27a.(\forall V3y \in A\_27b.((\neg (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFDOM}\ A\_27a\ A\_27b)\ V1f)))) \Rightarrow (p\ (ap\ V0P\ (ap\ (ap\ (c\_2Efinite\_\_map\_2EFUPDATE\ A\_27a\ A\_27b)\ V1f)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2x)\ V3y)))))))))) \Rightarrow (\forall V4f \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b).(p\ (ap\ V0P\ V4f))))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\neg(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(\forall V2s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V1y)\ V2s))) \Leftrightarrow ((V0x = V1y) \lor (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ V2s))))))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESING}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V0x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))))) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V0s)) \land (p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V1t)))))) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESING}\ A\_27a)\ V0s)) \Rightarrow (p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V0s)))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27a)\ (\lambda V0x \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V0x)\ \mathsf{c\_2Ebool\_2EF}))) = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0y \in A\_27a.((ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27a)\ (\lambda V1x \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V1x)\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27a)\ V1x)\ V0y)))) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V0y)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)))) \tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27a)\ (\lambda V2x \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V2x)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_5C\_2F}\ (ap\ V0P\ V2x))\ (ap\ V1Q\ V2x))))) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27a)\ (\lambda V3x \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V3x)\ (ap\ V0P\ V3x)))))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27a)\ (\lambda V4x \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V4x)\ (ap\ V1Q\ V4x)))))))) \tag{34}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0g \in (A\_27b^{A\_27a}).((\forall V1x \in A\_27a.\ (\forall V2y \in A\_27a.(((ap\ V0g\ V1x) = (ap\ V0g\ V2y)) \Leftrightarrow (V1x = V2y)))) \Rightarrow (\forall V3f \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27b\ A\_27c).(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27a)\ (\lambda V4x \in A\_27a.\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V4x)\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ (ap\ V0g\ V4x))\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFDOM}\ A\_27b\ A\_27c)\ V3f))))))))))$$