

thm_2Efinite__map_2EFLOOKUP__EMPTY
(TMZhHHRxeV-
GoX7uVum16LPoynAp4Gr2tJBo)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 8 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone$

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \tag{3}$$

Definition 10 We define c_Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_Esum_2EABS$

Let $ty_2Efinite_map_2E fmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty_2Efinite_map_2E fmap A0 A1) \quad (4)$$

Let $c_2Efinite_map_2E fmap_ABS : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Efinite_map_2E fmap_ABS A_27a A_27b \in ((ty_2Efinite_map_2E fmap A_27a A_27b)^{(ty_2Esum_2Esum A_27b ty_2Eone_2Eone)^{A_27a}}) \quad (5)$$

Definition 11 We define $c_2Efinite_map_2EFEMPTY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (ap (c_2Efinite_map_2E$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (6)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (7)$$

Definition 12 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_ABS A_27a) ($

Let $c_2Efinite_map_2E fmap_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Efinite_map_2E fmap_REP A_27a A_27b \in (((ty_2Esum_2Esum A_27b ty_2Eone_2Eone)^{A_27a})^{(ty_2Efinite_map_2E fmap A_27a A_27b)}) \quad (8)$$

Let $c_2Esum_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Esum_2EOUTL A_27a A_27b \in (A_27a^{(ty_2Esum_2Esum A_27a A_27b)}) \quad (9)$$

Definition 13 We define $c_2Efinite_map_2EFAPPLY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map$

Definition 14 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap (c_2Esum_2EABS$

Definition 15 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap (c_2Eoption_2Eoption$

Let $c_2Esum_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Esum_2EISL A_27a A_27b \in (2^{(ty_2Esum_2Esum A_27a A_27b)}) \quad (10)$$

Definition 16 We define $c_2Efinite_map_2EFDOM$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map$

Definition 17 We define `c_2Ebool_2ECOND` to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 18 We define `c_2Ebool_2EIN` to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x))$

Definition 19 We define `c_2Efinite_map_2EFLOOKUP` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_r$

Definition 20 We define `c_2Epred_set_2EEMPTY` to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Assume the following.

$$True \tag{11}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \tag{13}$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{14}$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{15}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{16}$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\ & V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27) \\ & V5y_27)))))))))) \tag{17} \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in \\ & A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap \\ & (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V2t1)\ V3t2) = V3t2)))) \tag{18} \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (ap\ (c_2Efinite_map_2EFDOM\ A_27a\ A_27b)\ (c_2Efinite_map_2EFEMPTY \\ & \quad A_27a\ A_27b)) = (c_2Epred_set_2EEMPTY\ A_27a)) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg (p\ (ap\ (ap \\ & (c_2Ebool_2EIN\ A_27a)\ V0x)\ (c_2Epred_set_2EEMPTY\ A_27a)))))) \end{aligned} \quad (20)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0k \in A_27b. ((ap\ (ap\ (c_2Efinite_map_2EFLOOKUP\ A_27b \\ & A_27a)\ (c_2Efinite_map_2EFEMPTY\ A_27b\ A_27a))\ V0k) = (c_2Eoption_2ENONE \\ & \quad A_27a))) \end{aligned}$$