

thm_2Efinite__map_2EFLOOKUP__FUNION (TMHbPSkggcrB99Uvigffe6YijTkAdtXydHc)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Efinite_map_2E fmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efinite_map_2E fmap A0 A1) \quad (1)$$

Let $c_2Efinite_map_2EFUNION : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2EFUNION A_27a A_27b \in (((ty_2Efinite_map_2E fmap A_27a A_27b)^{(ty_2Efinite_map_2E fmap A_27a A_27b)})^{(ty_2Efinite_map_2E fmap A_27a A_27b)}) \quad (2)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty ty_2Eone_2Eone \quad (3)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Esum_2Esum A0 A1) \quad (4)$$

Let $c_2Efinite_map_2E fmap_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2E fmap_REP A_27a A_27b \in (((ty_2Esum_2Esum A_27b ty_2Eone_2Eone)^{A_27a})^{(ty_2Efinite_map_2E fmap A_27a A_27b)}) \quad (5)$$

Let $c_2Esum_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{gathered} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EOUTL \\ A_27a\ A_27b \in (A_27a^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \end{gathered} \quad (6)$$

Definition 6 We define $c_2Efinite_map_2EFAPPLY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map_2EFAPPLY\ A_27a\ A_27b)$

Let $c_2Esum_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{gathered} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EISL \\ A_27a\ A_27b \in (2^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \end{gathered} \quad (7)$$

Definition 7 We define $c_2Efinite_map_2EFDOM$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map_2EFDOM\ A_27a\ A_27b)$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_2F_5C\ 2)\ (\lambda V2t \in 2. (ap\ (c_2Ebool_2E_2F_5C\ 2)\ V2t))))$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \mathbf{if}\ (\exists x \in A. p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x. x \in A \wedge p\ x))$ of type $\iota \Rightarrow \iota$.

Definition 10 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (ap\ (c_2Ebool_2ECOND\ V0t)\ V2t2))))$

Definition 11 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone))\ (\lambda V0x \in ty_2Eone_2Eone)$

Definition 12 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ (c_2Emin_2E_40\ V0t)\ c_2Ebool_2E_7E)))$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{gathered} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum \\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \end{gathered} \quad (8)$$

Definition 13 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (9)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{gathered} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in \\ ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \end{gathered} \quad (10)$$

Definition 14 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ (V0x \in ty_2Eoption_2Eoption\ A_27a))$

Definition 15 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Definition 16 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0x)$

Definition 17 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Definition 18 We define $c_2Efinite_map_2EFLOOKUP$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map_2EFLOOKUP)$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2Eoption_CASE \\ A_27a\ A_27b \in (((A_27b^{(A_27b^{A_27a})})^{A_27b})^{(ty_2Eoption_2Eoption\ A_27a)}) \end{aligned} \quad (11)$$

Definition 19 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2.))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod \\ A0\ A1) \end{aligned} \quad (12)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (13)$$

Definition 20 We define c_2Epair_2E2C to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2E2C))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (14)$$

Definition 21 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EUNION))$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee \neg(p\ V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(\neg(p\ V0t) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (20)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in A_27a.(((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2ET) \ V0t1) \ V1t2) = V0t1) \wedge ((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2EF) \ V0t1) \ V1t2) = V1t2)))))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \vee (\neg(p \ V1B)))) \wedge ((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \wedge (\neg(p \ V1B))))))) \quad (26)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A_27a.(\forall V3x_27 \in A_27a.(\forall V4y \in A_27a.(\forall V5y_27 \in A_27a.(((p \ V0P) \Leftrightarrow (p \ V1Q)) \wedge (((p \ V1Q) \Rightarrow (V2x = V3x_27)) \wedge ((\neg(p \ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ V0P) \ V2x) \ V4y) = (ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ V1Q) \ V3x_27) \ V5y_27))))))))) \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in \\ & A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap \\ & (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V2t1)\ V3t2) = V3t2)))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in (ty_2Efinite_map_2E fmap\ A_27a\ A_27b). (\forall V1g \in \\ & (ty_2Efinite_map_2E fmap\ A_27a\ A_27b). (((ap\ (c_2Efinite_map_2EFDOM \\ & A_27a\ A_27b)\ (ap\ (ap\ (c_2Efinite_map_2EFUNION\ A_27a\ A_27b)\ V0f)\ \\ & V1g)) = (ap\ (ap\ (c_2Epred_set_2EUNION\ A_27a)\ (ap\ (c_2Efinite_map_2EFDOM \\ & A_27a\ A_27b)\ V0f)))\ (ap\ (c_2Efinite_map_2EFDOM\ A_27a\ A_27b)\ V1g)))) \wedge \\ & (\forall V2x \in A_27a. ((ap\ (ap\ (c_2Efinite_map_2EFAPPLY\ A_27a \\ & A_27b)\ (ap\ (ap\ (c_2Efinite_map_2EFUNION\ A_27a\ A_27b)\ V0f)\ V1g)) \\ & V2x) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27b)\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V2x)\ (ap\ (c_2Efinite_map_2EFDOM\ A_27a\ A_27b)\ V0f))))\ (ap \\ & (ap\ (c_2Efinite_map_2EFAPPLY\ A_27a\ A_27b)\ V0f)\ V2x)))\ (ap\ (ap\ (\\ & c_2Efinite_map_2EFAPPLY\ A_27a\ A_27b)\ V1g)\ V2x)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (\forall V0v \in A_27b. (\forall V1f \in (A_27b^{A_27a}). ((ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\ & A_27a\ A_27b)\ (c_2Eoption_2ENONE\ A_27a)\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\ & A_27a. (\forall V3v \in A_27b. (\forall V4f \in (A_27b^{A_27a}). ((ap\ (ap \\ & (ap\ (c_2Eoption_2Eoption_CASE\ A_27a\ A_27b)\ (ap\ (c_2Eoption_2ESOME \\ & A_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ & A_27a. (((ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x) = (ap\ (c_2Eoption_2ESOME \\ & A_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg((c_2Eoption_2ENONE \\ & A_27a) = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x)))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\ & V2x)\ (ap\ (ap\ (c_2Epred_set_2EUNION\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\ & (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V2x)\ V1t)))))) \end{aligned} \quad (34)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f1 \in (ty_2Efinite_map_2Efmap\ A_27b\ A_27a).(\forall V1f2 \in \\ & (ty_2Efinite_map_2Efmap\ A_27b\ A_27a).(\forall V2k \in A_27b.(\\ (ap\ (ap\ (c_2Efinite_map_2EFLOOKUP\ A_27b\ A_27a)\ (ap\ (ap\ (c_2Efinite_map_2EFUNION \\ A_27b\ A_27a)\ V0f1)\ V1f2))\ V2k) = (ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\ A_27a\ (ty_2Eoption_2Eoption\ A_27a))\ (ap\ (ap\ (c_2Efinite_map_2EFLOOKUP \\ A_27b\ A_27a)\ V0f1)\ V2k))\ (ap\ (ap\ (c_2Efinite_map_2EFLOOKUP\ A_27b \\ A_27a)\ V1f2)\ V2k))\ (\lambda V3v \in A_27a.(ap\ (c_2Eoption_2ESOME\ A_27a) \\ V3v))))))))) \end{aligned}$$