

# thm\_2Efinite\_\_map\_2EFLOOKUP\_\_o\_\_f (TMPs-DZKHvBNdJ9iL28ge48AbS9YnEYPVUS9)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

**Definition 6** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 7** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap (c\_2Emin\_2E\_40$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

**Definition 11** We define  $c\_2Eone\_2Eone$  to be  $(ap (c\_2Emin\_2E\_40 ty\_2Eone\_2Eone) (\lambda V0x \in ty\_2Eone\_2Eone.V0x))$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \tag{3}$$

**Definition 12** We define  $c\_Esum\_2EINR$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27b. (ap (c\_Esum\_2EABS$   
 Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Eoption\_2Eoption A0) \quad (4)$$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS A\_27a \in ((ty\_2Eoption\_2Eoption A\_27a)^{(ty\_2Esum\_2Esum A\_27a ty\_2Eone\_2Eone)}) \quad (5)$$

**Definition 13** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota. (ap (c\_2Eoption\_2Eoption\_ABS A\_27a) ($   
 Let  $ty\_2Efinite\_map\_2E fmap : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty\_2Efinite\_map\_2E fmap A0 A1) \quad (6)$$

Let  $c\_2Efinite\_map\_2E fmap\_REP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow \forall A\_27b. nonempty A\_27b \Rightarrow c\_2Efinite\_map\_2E fmap\_REP A\_27a A\_27b \in (((ty\_2Esum\_2Esum A\_27b ty\_2Eone\_2Eone)^{A\_27a})^{(ty\_2Efinite\_map\_2E fmap A\_27a A\_27b)}) \quad (7)$$

Let  $c\_2Esum\_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow \forall A\_27b. nonempty A\_27b \Rightarrow c\_2Esum\_2EOUTL A\_27a A\_27b \in (A\_27a)^{(ty\_2Esum\_2Esum A\_27a A\_27b)} \quad (8)$$

**Definition 14** We define  $c\_2Efinite\_map\_2EFAPPLY$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (ty\_2Efinite\_map$

**Definition 15** We define  $c\_2Esum\_2EINL$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27a. (ap (c\_2Esum\_2EABS$

**Definition 16** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. (ap (c\_2Eoption\_2Eoption$

Let  $c\_2Esum\_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow \forall A\_27b. nonempty A\_27b \Rightarrow c\_2Esum\_2EISL A\_27a A\_27b \in (2^{(ty\_2Esum\_2Esum A\_27a A\_27b)}) \quad (9)$$

**Definition 17** We define  $c\_2Efinite\_map\_2EFDOM$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (ty\_2Efinite\_map$

**Definition 18** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. (\lambda V1f \in (2^{A\_27a}). (ap V1f V0x)))$

**Definition 19** We define  $c\_2Efinite\_map\_2EFLOOKUP$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (ty\_2Efinite\_map$

Let  $c\_2Efinite\_map\_2Eo\_f : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ & nonempty\ A\_27c \Rightarrow c\_2Efinite\_map\_2Eo\_f\ A\_27a\ A\_27b\ A\_27c \in ((( \\ & ty\_2Efinite\_map\_2Efmap\ A\_27a\ A\_27c)^{(ty\_2Efinite\_map\_2Efmap\ A\_27a\ A\_27b)})^{(A\_27c^{A\_27b})}) \end{aligned} \quad (10)$$

Let  $c\_2Eoption\_2Eoption\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2Eoption\_CASE \\ & A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \end{aligned} \quad (11)$$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (13)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \vee \neg(p\ V0t))) \quad (15)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow \\ & True)) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in \\ & A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\ & p\ V0t)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ & A\_27a. (((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1) \\ & V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF) \\ & V0t1)\ V1t2) = V1t2)))))) \end{aligned} \quad (19)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\ & (\forall V5y\_27 \in A\_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap (ap (ap (c\_2Ebool\_2ECOND\ A\_27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c\_2Ebool\_2ECOND\ A\_27a) V1Q) V3x\_27) \\ & V5y\_27))))))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ & A\_27a. ((ap (ap (ap (c\_2Ebool\_2ECOND\ A\_27a) c\_2Ebool\_2ET) V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a. (\forall V3t2 \in A\_27a. ((ap \\ & (ap (ap (c\_2Ebool\_2ECOND\ A\_27a) c\_2Ebool\_2EF) V2t1) V3t2) = V3t2)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ & nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27c^{A\_27b}). (\forall V1g \in (ty\_2Efinite\_map\_2E fmap \\ & A\_27a\ A\_27b). ((ap (c\_2Efinite\_map\_2EFDOM\ A\_27a\ A\_27c) (ap (ap \\ & (c\_2Efinite\_map\_2Eo\_f\ A\_27a\ A\_27b\ A\_27c) V0f) V1g)) = (ap (c\_2Efinite\_map\_2EFDOM \\ & A\_27a\ A\_27b) V1g)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ & nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27c^{A\_27b}). (\forall V1g \in (ty\_2Efinite\_map\_2E fmap \\ & A\_27a\ A\_27b). (\forall V2x \in A\_27a. ((p (ap (ap (c\_2Ebool\_2EIN\ A\_27a) \\ & V2x) (ap (c\_2Efinite\_map\_2EFDOM\ A\_27a\ A\_27b) V1g))) \Rightarrow ((ap (ap \\ & (c\_2Efinite\_map\_2EFAPPLY\ A\_27a\ A\_27c) (ap (ap (c\_2Efinite\_map\_2Eo\_f \\ & A\_27a\ A\_27b\ A\_27c) V0f) V1g)) V2x) = (ap V0f (ap (ap (c\_2Efinite\_map\_2EFAPPLY \\ & A\_27a\ A\_27b) V1g) V2x))))))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & (\forall V0v \in A\_27b. (\forall V1f \in (A\_27b^{A\_27a}). ((ap (ap (ap (c\_2Eoption\_2Eoption\_CASE \\ & A\_27a\ A\_27b) (c\_2Eoption\_2ENONE\ A\_27a)) V0v) V1f) = V0v))) \wedge (\forall V2x \in \\ & A\_27a. (\forall V3v \in A\_27b. (\forall V4f \in (A\_27b^{A\_27a}). ((ap (ap \\ & (ap (c\_2Eoption\_2Eoption\_CASE\ A\_27a\ A\_27b) (ap (c\_2Eoption\_2ESOME \\ & A\_27a) V2x)) V3v) V4f) = (ap V4f V2x))))))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in \\ & A.27a. (((ap\ (c.2Eoption\_2ESOME\ A.27a)\ V0x) = (ap\ (c.2Eoption\_2ESOME \\ & A.27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (26)$$

**Theorem 1**

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ & nonempty\ A.27c \Rightarrow (\forall V0f \in (A.27a^{A.27c}). (\forall V1fm \in (ty\_2Efinite\_map\_2E fmap \\ & A.27b\ A.27c). (\forall V2k \in A.27b. ((ap\ (ap\ (c.2Efinite\_map\_2EFLOOKUP \\ & A.27b\ A.27a)\ (ap\ (ap\ (c.2Efinite\_map\_2Eo\_f\ A.27b\ A.27c\ A.27a) \\ & V0f)\ V1fm))\ V2k) = (ap\ (ap\ (ap\ (c.2Eoption\_2Eoption\_CASE\ A.27c \\ & (ty\_2Eoption\_2Eoption\ A.27a))\ (ap\ (ap\ (c.2Efinite\_map\_2EFLOOKUP \\ & A.27b\ A.27c)\ V1fm)\ V2k))\ (c.2Eoption\_2ENONE\ A.27a))\ (\lambda V3v \in \\ & A.27c. (ap\ (c.2Eoption\_2ESOME\ A.27a)\ (ap\ V0f\ V3v)))))))))) \end{aligned}$$