

# thm\_2Efinite\_map\_2EFMEQ\_SINGLE\_SIMPLE\_ELIM (TMa5pe4MxVat7KGBzUCaTcWXuSPbRrq5R8)

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**Definition 1** We define `c_2Emin_2E_40` to be  $\lambda A.\lambda P \in 2^A.$ **if**  $(\exists x \in A.p (ap P x))$  **then** *(the*  $(\lambda x.x \in A \wedge p$   
of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Emin_2E_3D` to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$   
of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define `c_2Ebool_2E_3F` to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A$

**Definition 4** We define `c_2Ebool_2E_T` to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1$

Let `ty_2Eone_2Eone` :  $\iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

**Definition 5** We define `c_2Eone_2Eone` to be  $(ap (c_2Emin_2E_40 ty\_2Eone\_2Eone) (\lambda V0x \in ty\_2Eone\_2E$

**Definition 6** We define `c_2Ebool_2E_21` to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27$

**Definition 7** We define `c_2Ebool_2E_F` to be  $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 8** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$   
of type  $\iota$ .

**Definition 9** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_F$

**Definition 10** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Let `ty_2Esum_2Esum` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Let `c_2Esum_2EABS_sum` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c\_2Esum\_2EABS\_sum\ A.27a\ A.27b \in ((ty\_2Esum\_2Esum\ A.27a\ A.27b)^{((2^{A-27b})^{A-27a})^2}) \tag{3}$$

**Definition 11** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap (c\_2Esum\_2EABS$

Let  $ty\_2Efinite\_map\_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Efinite\_map\_2Efmap A0 A1) \quad (4)$$

Let  $c\_2Efinite\_map\_2Efmap\_ABS : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Efinite\_map\_2Efmap\_ABS A\_27a A\_27b \in ((ty\_2Efinite\_map\_2Efmap A\_27a A\_27b)^{(ty\_2Esum\_2Esum A\_27b ty\_2Eone\_2Eone)^{A\_27a}}) \quad (5)$$

**Definition 12** We define  $c\_2Efinite\_map\_2EFEMPTY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(ap (c\_2Efinite\_map\_2E$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (6)$$

Let  $c\_2Efinite\_map\_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Efinite\_map\_2EFUPDATE A\_27a A\_27b \in (((ty\_2Efinite\_map\_2Efmap A\_27a A\_27b)^{(ty\_2Epair\_2Eprod A\_27a A\_27b)})^{(ty\_2Efinite\_map\_2E$$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (8)$$

Let  $c\_2Elist\_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Elist\_2EFOLDL A\_27a A\_27b \in (((A\_27b^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27b})^{(A\_27b^{A\_27a})^{A\_27b}}) \quad (9)$$

**Definition 13** We define  $c\_2Efinite\_map\_2EFUPDATE\_LIST$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(ap (c\_2Elist\_2E$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (10)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (11)$$

Let  $c\_2Elist\_2ELIST\_TO\_SET : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ELIST\_TO\_SET A\_27a \in ((2^{A\_27a})^{(ty\_2Elist\_2Elist A\_27a)}) \quad (12)$$

**Definition 14** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. (\lambda V1f \in (2^{A\_27a}). (ap\ V1f\ V0x)))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (13)$$

**Definition 15** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap\ (c\_2Ebool\_2E\_21\ 2)\ V0x\ V1y)$

**Definition 16** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ V0t1\ V1t2)))$

Assume the following.

$$True \quad (14)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2))))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \vee \neg(p\ V0t))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. (((p\ V0t) \Rightarrow False) \Rightarrow \neg(p\ V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (\neg(p\ V0t) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (20) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee \\ & (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (21) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ & (p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (22) \end{aligned}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (23)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (24)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (26)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((p V0P) \wedge (\forall V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A.27a.((p V0P) \wedge (p (ap V1Q V3x))))))) \quad (27)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((\forall V2x \in A.27a.((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a.(p (ap V1Q V3x))))))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C))))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge (\neg(p V1B))))))) \quad (31)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0f \in (ty\_2Efinite\_map\_2E fmap\ A\_27a\ A\_27b).(((ap\ (ap \\
& \quad (c\_2Efinite\_map\_2EFUPDATE\_LIST\ A\_27a\ A\_27b)\ V0f)\ (c\_2Elist\_2ENIL \\
& \quad (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)))) = V0f) \wedge (\forall V1h \in (ty\_2Epair\_2Eprod \\
& \quad A\_27a\ A\_27b).(\forall V2t \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod \\
& \quad A\_27a\ A\_27b)).((ap\ (ap\ (c\_2Efinite\_map\_2EFUPDATE\_LIST\ A\_27a \\
& \quad A\_27b)\ V0f)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)) \\
& \quad V1h)\ V2t))) = (ap\ (ap\ (c\_2Efinite\_map\_2EFUPDATE\_LIST\ A\_27a\ A\_27b) \\
& \quad (ap\ (ap\ (c\_2Efinite\_map\_2EFUPDATE\ A\_27a\ A\_27b)\ V0f)\ V1h))\ V2t)))))) \\
& \hspace{15em} (32)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0f1 \in (ty\_2Efinite\_map\_2E fmap\ A\_27a\ A\_27b).(\forall V1f2 \in \\
& \quad (ty\_2Efinite\_map\_2E fmap\ A\_27a\ A\_27b).(\forall V2k \in A\_27a.( \\
& \quad \forall V3v1 \in A\_27b.(\forall V4v2 \in A\_27b.(((ap\ (ap\ (c\_2Efinite\_map\_2EFUPDATE \\
& \quad A\_27a\ A\_27b)\ V0f1)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V2k)\ V3v1)) = \\
& \quad (ap\ (ap\ (c\_2Efinite\_map\_2EFUPDATE\ A\_27a\ A\_27b)\ V1f2)\ (ap\ (ap\ ( \\
& \quad c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V2k)\ V4v2)))) \Rightarrow ((V3v1 = V4v2) \wedge (\forall V5v \in \\
& \quad A\_27b.((ap\ (ap\ (c\_2Efinite\_map\_2EFUPDATE\ A\_27a\ A\_27b)\ V0f1) \\
& \quad (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V2k)\ V5v)) = (ap\ (ap\ (c\_2Efinite\_map\_2EFUPDATE \\
& \quad A\_27a\ A\_27b)\ V1f2)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V2k)\ V5v)))))))))) \\
& \hspace{15em} (33)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0f1 \in (ty\_2Efinite\_map\_2E fmap\ A\_27a\ A\_27b).(\forall V1kvl \in \\
& \quad (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)).(\forall V2p \in \\
& \quad (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(((ap\ (ap\ (c\_2Efinite\_map\_2EFUPDATE \\
& \quad A\_27a\ A\_27b)\ V0f1)\ V2p) = (ap\ (ap\ (c\_2Efinite\_map\_2EFUPDATE\_LIST \\
& \quad A\_27a\ A\_27b)\ (c\_2Efinite\_map\_2EFEMPTY\ A\_27a\ A\_27b))\ V1kvl)) \Rightarrow \\
& \quad (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ V2p) \\
& \quad (ap\ (c\_2Elist\_2ELIST\_TO\_SET\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)) \\
& \quad V1kvl)))))) \\
& \hspace{15em} (34)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0x \in A\_27a.((p\ (ap\ (ap \\
& \quad (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ (ap\ (c\_2Elist\_2ELIST\_TO\_SET\ A\_27a) \\
& \quad (c\_2Elist\_2ENIL\ A\_27a)))) \Leftrightarrow False)) \wedge (\forall V1x \in A\_27a.(\forall V2h \in \\
& \quad A\_27a.(\forall V3t \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\
& \quad A\_27a)\ V1x)\ (ap\ (c\_2Elist\_2ELIST\_TO\_SET\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
& \quad A\_27a)\ V2h)\ V3t)))) \Leftrightarrow ((V1x = V2h) \vee (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a) \\
& \quad V1x)\ (ap\ (c\_2Elist\_2ELIST\_TO\_SET\ A\_27a)\ V3t)))))) \\
& \hspace{15em} (35)
\end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0x \in A\_27a. (\forall V1y \in A\_27b. (\forall V2a \in A\_27a. (\forall V3b \in \\ & A\_27b. (((ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap \\ & (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \\ & \end{aligned} \quad (36)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (38)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \quad (39) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \quad (40) \end{aligned}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (41)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ( \\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ & (p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \quad (42) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ( \\ & (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\ & (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \quad (43) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ( \\ & (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\ & ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \quad (44) \end{aligned}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\neg(p V1q)) \vee ((p V2r) \vee \neg(p V0p)))))))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (\neg(p V1q)) \vee \neg(p V0p)))))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow \neg(p V1q))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V0p))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V1q))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2. (\neg(\neg(p V0p))) \Rightarrow (p V0p)) \quad (51)$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0P \in (2^{(ty\_2Efinite\_map\_2E fmap\ A\_27a\ A\_27b)}). (\forall V1k \in \\ & A\_27a. (\forall V2v \in A\_27b. (\forall V3ck \in A\_27a. (\forall V4cv \in \\ & A\_27b. (\forall V5nv \in A\_27b. ((\exists V6fm \in (ty\_2Efinite\_map\_2E fmap \\ & A\_27a\ A\_27b). (((ap\ (ap\ (c\_2Efinite\_map\_2EFUPDATE\ A\_27a\ A\_27b) \\ & V6fm)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V1k)\ V2v)) = (ap\ (ap\ (c\_2Efinite\_map\_2EFUPDATE \\ & A\_27a\ A\_27b)\ (c\_2Efinite\_map\_2EFEMPTY\ A\_27a\ A\_27b))\ (ap\ (ap\ ( \\ & c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V3ck)\ V4cv))) \wedge (p\ (ap\ V0P\ (ap\ (ap\ (c\_2Efinite\_map\_2EFUPDATE \\ & A\_27a\ A\_27b)\ V6fm)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V1k)\ V5nv)))))) \Leftrightarrow \\ & ((V1k = V3ck) \wedge ((V2v = V4cv) \wedge (p\ (ap\ V0P\ (ap\ (ap\ (c\_2Efinite\_map\_2EFUPDATE \\ & A\_27a\ A\_27b)\ (c\_2Efinite\_map\_2EFEMPTY\ A\_27a\ A\_27b))\ (ap\ (ap\ ( \\ & c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V3ck)\ V5nv)))))))))) \end{aligned}$$