thm_2Efinite__map_2EFMERGE__DOMSUB (TMVoHcWw3h3End1AeGVZiihB4p1s1h4i7u7)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $ty\_2Efinite\_\_map\_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efinite\_\_map\_2Efmap\ A0\ A1) \quad (1)$$

Let $c\_2Efinite\_\_map\_2EFMERGE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_\_map\_2 \quad A\_27a\ A\_27b \in ((((ty\_2Efinite\_\_map\_2Efmap\ A\_27b\ A\_27a)^{(ty\_2Efinite\_\_map\_2Efmap\ A\_27b\ A\_27a)})^{(ty\_2Efi} \quad (2)$$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2EF})$.

Definition 6 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 7 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 8 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (3)$$

Let $c\_2Epair\_2EABS\_\_prod : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (4)$$

Definition 10 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \quad (5)$$

Definition 11 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 12 *We define* c_2Epred__set_2EUNIV *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 13 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 14 *We define* c_2Epred__set_2EDIFF *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2$

Definition 15 *We define* c_2Epred__set_2ECOMPL *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{A\_27a}).(ap\ (ap$ (c_2Epred__set

Let $c\_2Efinite\_\_map\_2EDRESTRICT : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Efinite\_\_map\_2EDRESTRICT\ A\_27a\ A\_27b \in (((ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b)^{(2^{A\_27a})})^{(ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b)}) \quad (6)$$

Definition 16 *We define* c_2Efinite__map_2Efdomsub *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0fm \in (ty\_2Efinite\_\_m$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.((V0x = V0x){\Leftrightarrow}True)) \quad (8)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}(\forall V0f \in ((A\_27b^{A\_27b})^{A\_27b}).(\forall V1st1 \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b).(\forall V2st2 \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b).(\forall V3vs \in (2^{A\_27a}).((ap\ (ap\ (c\_2Efinite\_\_map\_2EDRESTRICT\ A\_27a\ A\_27b)\ (ap\ (ap\ (ap\ (c\_2Efinite\_\_map\_2EFMERGE\ A\_27b\ A\_27a)\ V0f)\ V1st1)\ V2st2))\ V3vs) = (ap\ (ap\ (ap\ (c\_2Efinite\_\_map\_2EFMERGE\ A\_27b\ A\_27a)\ V0f)\ (ap\ (ap\ (c\_2Efinite\_\_map\_2EDRESTRICT\ A\_27a\ A\_27b)\ V1st1)\ V3vs))\ (ap\ (ap\ (c\_2Efinite\_\_map\_2EDRESTRICT\ A\_27a\ A\_27b)\ V2st2)\ V3vs))))))) \quad (9)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0m \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V1m1 \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27b\ A\_27a).(\forall V2m2 \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27b\ A\_27a).(\forall V3k \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2Efdomsub}\ A\_27b\ A\_27a)\ (ap\ (ap\ (ap\ (c\_2Efinite\_\_map\_2EFMERGE\ A\_27a\ A\_27b)\ V0m)\ V1m1)\ V2m2))\ V3k) = (ap\ (ap\ (ap\ (c\_2Efinite\_\_map\_2EFMERGE\ A\_27a\ A\_27b)\ V0m)\ (ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2Efdomsub}\ A\_27b\ A\_27a)\ V1m1)\ V3k))\ (ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2Efdomsub}\ A\_27b\ A\_27a)\ V2m2)\ V3k)))))))$$