

thm_2Efinite_map_2EFOLDL2_FUPDATE_LIST (TMFZEGy7v54V3ukhT7d6hDFNqFrsWRcAtc8)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Efinite_map_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efinite_map_2Efmap A0 A1) \quad (1)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (2)$$

Let $c_2Efinite_map_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2EFUPDATE A_27a A_27b \in (((ty_2Efinite_map_2Efmap A_27a A_27b)^{(ty_2Epair_2Eprod A_27a A_27b)})^{(ty_2Efinite_map_2EFUPDATE A_27a A_27b)}) \quad (3)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (4)$$

Let $c_2Elist_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EFOLDL \\ & A_27a\ A_27b \in (((A_27b^{(ty_2Elist_2Elist\ A_27a)})^{A_27b})^{((A_27b^{A_27a})^{A_27b})}) \end{aligned} \quad (5)$$

Definition 7 We define $c_2Efinite_map_2EFUPDATE_LIST$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(ap\ (c_2Elist_2EFOLDL\ A_27a)\ A_27b)$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (V1t2\ t1))))$

Let $c_2Elist_2EMAP2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow c_2Elist_2EMAP2\ A_27a\ A_27b\ A_27c \in (((ty_2Elist_2Elist \\ & A_27a)^{(ty_2Elist_2Elist\ A_27c)})^{(ty_2Elist_2Elist\ A_27b)})^{((A_27a^{A_27c})^{A_27b})} \end{aligned} \quad (6)$$

Let $c_2Elist_2EFOLDL2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow c_2Elist_2EFOLDL2\ A_27a\ A_27b\ A_27c \in (((A_27a^{(ty_2Elist_2Elist\ A_27c)})^{(ty_2Elist_2Elist\ A_27b)})^{A_27c}) \end{aligned} \quad (7)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (8)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \quad (9)$$

Let $c_2Elist_2EZIP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EZIP \\ & A_27a\ A_27b \in ((ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b))^{(ty_2Epair_2Eprod\ (ty_2Elist_2Elist\ A_27a)\ A_27b)}) \end{aligned} \quad (10)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ & A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (11)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ & A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (12)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ & A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{((A_27b^{A_27a})^{A_27b})}) \end{aligned} \quad (13)$$

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Definition 10 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EMAP \\ & A_27a\ A_27b \in (((ty_2Elist_2Elist\ A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{(A_27b^{A_27a})}) \end{aligned} \quad (14)$$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\ & 2.(((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0l \in (ty_2Elist_2Elist\ A_27a).(\forall V1f \in (A_27b^{A_27a}). \\ & ((ap (c_2Elist_2ELENGTH\ A_27b) (ap (ap (c_2Elist_2EMAP\ A_27a\ A_27b) \\ & V1f)\ V0l)) = (ap (c_2Elist_2ELENGTH\ A_27a)\ V0l)))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0l1 \in (ty_2Elist_2Elist\ A_27a).(\forall V1l2 \in (ty_2Elist_2Elist \\
& \quad A_27b).(((ap\ (c_2Elist_2ELENGTH\ A_27a)\ V0l1) = (ap\ (c_2Elist_2ELENGTH \\
& \quad A_27b)\ V1l2)) \Rightarrow (((ap\ (c_2Elist_2ELENGTH\ (ty_2Epair_2Eprod\ A_27a \\
& \quad A_27b))\ (ap\ (c_2Elist_2EZIP\ A_27a\ A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad (ty_2Elist_2Elist\ A_27a)\ (ty_2Elist_2Elist\ A_27b))\ V0l1)\ V1l2)))) = \\
& \quad (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V0l1)) \wedge ((ap\ (c_2Elist_2ELENGTH \\
& \quad (ty_2Epair_2Eprod\ A_27a\ A_27b))\ (ap\ (c_2Elist_2EZIP\ A_27a\ A_27b) \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Elist_2Elist\ A_27a)\ (ty_2Elist_2Elist \\
& \quad A_27b))\ V0l1)\ V1l2)))) = (ap\ (c_2Elist_2ELENGTH\ A_27b)\ V1l2)))))) \\
& \hspace{15em} (22)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0l1 \in (\\
& \quad ty_2Elist_2Elist\ A_27a).(\forall V1l2 \in (ty_2Elist_2Elist\ A_27b). \\
& \quad (\forall V2f1 \in (A_27c^{A_27a}).(\forall V3f2 \in (A_27d^{A_27b}).(((\\
& \quad ap\ (c_2Elist_2ELENGTH\ A_27a)\ V0l1) = (ap\ (c_2Elist_2ELENGTH\ A_27b) \\
& \quad V1l2)) \Rightarrow (((ap\ (c_2Elist_2EZIP\ A_27c\ A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad (ty_2Elist_2Elist\ A_27c)\ (ty_2Elist_2Elist\ A_27b))\ (ap\ (ap\ (c_2Elist_2EMAP \\
& \quad A_27a\ A_27c)\ V2f1)\ V0l1))\ V1l2)) = (ap\ (ap\ (c_2Elist_2EMAP\ (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27b)\ (ty_2Epair_2Eprod\ A_27c\ A_27b))\ (\lambda V4p \in (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27b).(ap\ (ap\ (c_2Epair_2E_2C\ A_27c\ A_27b)\ (ap\ V2f1\ (ap\ (\\
& \quad c_2Epair_2EFST\ A_27a\ A_27b)\ V4p))))\ (ap\ (c_2Epair_2ESND\ A_27a\ A_27b) \\
& \quad V4p))))\ (ap\ (c_2Elist_2EZIP\ A_27a\ A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad (ty_2Elist_2Elist\ A_27a)\ (ty_2Elist_2Elist\ A_27b))\ V0l1)\ V1l2)))))) \wedge \\
& \quad ((ap\ (c_2Elist_2EZIP\ A_27a\ A_27d)\ (ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Elist_2Elist \\
& \quad A_27a)\ (ty_2Elist_2Elist\ A_27d))\ V0l1)\ (ap\ (ap\ (c_2Elist_2EMAP \\
& \quad A_27b\ A_27d)\ V3f2)\ V1l2))) = (ap\ (ap\ (c_2Elist_2EMAP\ (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27b)\ (ty_2Epair_2Eprod\ A_27a\ A_27d))\ (\lambda V5p \in (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27b).(ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27d)\ (ap\ (c_2Epair_2EFST \\
& \quad A_27a\ A_27b)\ V5p))\ (ap\ V3f2\ (ap\ (c_2Epair_2ESND\ A_27a\ A_27b)\ V5p)))))) \\
& \quad (ap\ (c_2Elist_2EZIP\ A_27a\ A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Elist_2Elist \\
& \quad A_27a)\ (ty_2Elist_2Elist\ A_27b))\ V0l1)\ V1l2))))))))) \\
& \hspace{15em} (23)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow (\forall V0l1 \in (ty_2Elist_2Elist\ A.27a). (\forall V1l2 \in \\
& (ty_2Elist_2Elist\ A.27b). (((ap\ (c_2Elist_2ELENGTH\ A.27a)\ V0l1) = \\
& (ap\ (c_2Elist_2ELENGTH\ A.27b)\ V1l2)) \Rightarrow (\forall V2f \in ((A.27c^{A.27b})^{A.27a}). \\
& ((ap\ (ap\ (ap\ (c_2Elist_2EMAP2\ A.27c\ A.27a\ A.27b)\ V2f)\ V0l1)\ V1l2) = \\
& (ap\ (ap\ (c_2Elist_2EMAP\ (ty_2Epair_2Eprod\ A.27a\ A.27b)\ A.27c) \\
& (ap\ (c_2Epair_2EUNCURRY\ A.27a\ A.27b\ A.27c)\ V2f))\ (ap\ (c_2Elist_2EZIP \\
& A.27a\ A.27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Elist_2Elist\ A.27a) \\
& (ty_2Elist_2Elist\ A.27b))\ V0l1)\ V1l2)))))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow (\forall V0l1 \in (ty_2Elist_2Elist\ A.27a). (\forall V1l2 \in \\
& (ty_2Elist_2Elist\ A.27b). (((ap\ (c_2Elist_2ELENGTH\ A.27a)\ V0l1) = \\
& (ap\ (c_2Elist_2ELENGTH\ A.27b)\ V1l2)) \Rightarrow (\forall V2f \in (((A.27c^{A.27b})^{A.27a})^{A.27c}). \\
& (\forall V3a \in A.27c. ((ap\ (ap\ (ap\ (ap\ (c_2Elist_2EFOLDL2\ A.27c\ A.27a \\
& A.27b)\ V2f)\ V3a)\ V0l1)\ V1l2) = (ap\ (ap\ (ap\ (c_2Elist_2EFOLDL\ (ty_2Epair_2Eprod \\
& A.27a\ A.27b)\ A.27c)\ (\lambda V4a \in A.27c. (ap\ (c_2Epair_2EUNCURRY\ A.27a \\
& A.27b\ A.27c)\ (ap\ V2f\ V4a))))\ V3a)\ (ap\ (c_2Elist_2EZIP\ A.27a\ A.27b) \\
& (ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Elist_2Elist\ A.27a)\ (ty_2Elist_2Elist \\
& A.27b))\ V0l1)\ V1l2)))))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0ls \in (ty_2Elist_2Elist\ A.27a). (\forall V1f \in (A.27b^{(ty_2Epair_2Eprod\ A.27a\ A.27a)}). \\
& ((ap\ (ap\ (c_2Elist_2EMAP\ (ty_2Epair_2Eprod\ A.27a\ A.27a)\ A.27b) \\
& V1f)\ (ap\ (c_2Elist_2EZIP\ A.27a\ A.27a)\ (ap\ (ap\ (c_2Epair_2E_2C\ (\\
& ty_2Elist_2Elist\ A.27a)\ (ty_2Elist_2Elist\ A.27a))\ V0ls)\ V0ls))) = \\
& (ap\ (ap\ (c_2Elist_2EMAP\ A.27a\ A.27b)\ (\lambda V2x \in A.27a. (ap\ V1f\ (ap \\
& (ap\ (c_2Epair_2E_2C\ A.27a\ A.27a)\ V2x)\ V2x))))\ V0ls)))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0x \in A.27a. (\forall V1y \in A.27b. ((ap\ (c_2Epair_2EFST\ A.27a \\
& A.27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27b)\ V0x)\ V1y)) = V0x)))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0x \in A.27a. (\forall V1y \in A.27b. ((ap\ (c_2Epair_2ESND\ A.27a \\
& A.27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27b)\ V0x)\ V1y)) = V1y)))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& \quad nonempty\ A.27c \Rightarrow (\forall V0f \in ((A.27c^{A.27b})^{A.27a}). (\forall V1x \in \\
& \quad A.27a. (\forall V2y \in A.27b. ((ap\ (ap\ (c.2Epair_2EUNCURRY\ A.27a \\
& \quad A.27b\ A.27c)\ V0f)\ (ap\ (ap\ (c.2Epair_2E_2C\ A.27a\ A.27b)\ V1x)\ V2y))) = \\
& \quad (ap\ (ap\ V0f\ V1x)\ V2y))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& \quad nonempty\ A.27c \Rightarrow (\forall V0P \in (A.27c^{(ty_2Epair_2Eprod\ A.27a\ A.27b)}). \\
& \quad ((\lambda V1p \in (ty_2Epair_2Eprod\ A.27a\ A.27b). (ap\ V0P\ V1p)) = (ap\ (\\
& \quad c.2Epair_2EUNCURRY\ A.27a\ A.27b\ A.27c)\ (\lambda V2p1 \in A.27a. (\lambda V3p2 \in \\
& \quad A.27b. (ap\ V0P\ (ap\ (ap\ (c.2Epair_2E_2C\ A.27a\ A.27b)\ V2p1)\ V3p2))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& \quad nonempty\ A.27c \Rightarrow (\forall V0f \in ((A.27a^{A.27b})^{A.27a}). (\forall V1e \in \\
& \quad A.27a. (\forall V2g \in (A.27b^{A.27c}). (\forall V3l \in (ty_2Elist_2Elist \\
& \quad A.27c). ((ap\ (ap\ (ap\ (c.2Elist_2EFOLDL\ A.27b\ A.27a)\ V0f)\ V1e)\ (ap \\
& \quad (ap\ (c.2Elist_2EMAP\ A.27c\ A.27b)\ V2g)\ V3l))) = (ap\ (ap\ (ap\ (c.2Elist_2EFOLDL \\
& \quad A.27c\ A.27a)\ (\lambda V4x \in A.27a. (\lambda V5y \in A.27c. (ap\ (ap\ V0f\ V4x)\ (\\
& \quad ap\ V2g\ V5y))))))\ V1e)\ V3l))))))
\end{aligned} \tag{31}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& \quad nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow (\forall V0f1 \in (\\
& \quad (A.27c^{A.27b})^{A.27a}). (\forall V1f2 \in ((A.27d^{A.27b})^{A.27a}). (\forall V2bs \in \\
& \quad (ty_2Elist_2Elist\ A.27a). (\forall V3cs \in (ty_2Elist_2Elist\ A.27b). \\
& \quad (\forall V4a \in (ty_2Efinite_map_2E fmap\ A.27c\ A.27d). (((ap\ (c.2Elist_2ELENGTH \\
& \quad A.27a)\ V2bs) = (ap\ (c.2Elist_2ELENGTH\ A.27b)\ V3cs))) \Rightarrow ((ap\ (ap\ (ap \\
& \quad (ap\ (c.2Elist_2EFOLDL2\ (ty_2Efinite_map_2E fmap\ A.27c\ A.27d) \\
& \quad A.27a\ A.27b)\ (\lambda V5fm \in (ty_2Efinite_map_2E fmap\ A.27c\ A.27d). \\
& \quad (\lambda V6b \in A.27a. (\lambda V7c \in A.27b. (ap\ (ap\ (c.2Efinite_map_2EFUPDATE \\
& \quad A.27c\ A.27d)\ V5fm)\ (ap\ (ap\ (c.2Epair_2E_2C\ A.27c\ A.27d)\ (ap\ (ap\ V0f1 \\
& \quad V6b)\ V7c))\ (ap\ (ap\ V1f2\ V6b)\ V7c))))))\ V4a)\ V2bs)\ V3cs) = (ap\ (ap\ (\\
& \quad c.2Efinite_map_2EFUPDATE_LIST\ A.27c\ A.27d)\ V4a)\ (ap\ (c.2Elist_2EZIP \\
& \quad A.27c\ A.27d)\ (ap\ (ap\ (c.2Epair_2E_2C\ (ty_2Elist_2Elist\ A.27c) \\
& \quad (ty_2Elist_2Elist\ A.27d))\ (ap\ (ap\ (ap\ (c.2Elist_2EMAP2\ A.27c\ A.27a \\
& \quad A.27b)\ V0f1)\ V2bs)\ V3cs))\ (ap\ (ap\ (ap\ (c.2Elist_2EMAP2\ A.27d\ A.27a \\
& \quad A.27b)\ V1f2)\ V2bs)\ V3cs))))))))))
\end{aligned}$$