thm_2Efinite__map_2EFOLDL2__FUPDATE__LIST__paired (TMJXRui9CoUreqBbbRPxVW7j9et7aLHVxY6)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $ty\_2Efinite\_\_map\_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efinite\_\_map\_2Efmap\ A0\ A1) \quad (1)$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (2)$$

Let $c\_2Efinite\_\_map\_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_\_map\_2EFUPDAT\ A\_27a\ A\_27b \in (((ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})^{(ty\_2Efinite\_\_map} \quad (3)$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (4)$$

Let $c\_2Elist\_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EFOLDL\ A\_27a\ A\_27b \in (((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27b})^{((A\_27b^{A\_27a})^{A\_27b})}) \tag{5}$$

Definition 7 *We define* c_2Efinite__map_2EFUPDATE__LIST *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(ap\ (c\_2Elist\_2EF$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in 2$

Let $c\_2Elist\_2EMAP2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow c\_2Elist\_2EMAP2\ A\_27a\ A\_27b\ A\_27c \in ((((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27c)})^{(ty\_2Elist\_2Elist\ A\_27b)})^{((A\_27a^{A\_27c})^{A\_27b})}) \tag{6}$$

Let $c\_2Elist\_2EFOLDL2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow c\_2Elist\_2EFOLDL2\ A\_27a\ A\_27b\ A\_27c \in ((((A\_27a^{(ty\_2Elist\_2Elist\ A\_27c)})^{(ty\_2Elist\_2} \tag{7}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{8}$$

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{9}$$

Let $c\_2Elist\_2EZIP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EZIP\ A\_27a\ A\_27b \in ((ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))^{(ty\_2Epair\_2Eprod\ (ty\_2Elist\_2Elist\ A\_27} \tag{10}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{11}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{12}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{13}$$

Definition 9 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0x\in A\_27a.\lambda V1y\in A\_27b.(ap\ (c\_2E$

Definition 10 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda A\_27c:\iota.\lambda V0f\in((A\_27c^{A\_27}$

Let $c\_2Elist\_2EMAP:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Elist\_2EMAP\ A\_27a\ A\_27b\in(((ty\_2Elist\_2Elist\ A\_27b)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(A\_27b^{A\_27a})}) \tag{14}$$

Assume the following.

$$True \tag{15}$$

Assume the following.

$$(\forall V0t\in 2.(((True\Rightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Rightarrow True)\Leftrightarrow True)\wedge(((False\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge(((p\ V0t)\Rightarrow False)\Leftrightarrow(\neg(p\ V0t)))))))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x\in A\_27a.((V0x=V0x)\Leftrightarrow True)) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x\in A\_27a.(\forall V1y\in A\_27a.((V0x=V1y)\Leftrightarrow(V1y=V0x)))) \tag{18}$$

Assume the following.

$$(\forall V0t1\in 2.(\forall V1t2\in 2.(\forall V2t3\in 2.(((p\ V0t1)\Rightarrow((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{19}$$

Assume the following.

$$(\forall V0x\in 2.(\forall V1x\_27\in 2.(\forall V2y\in 2.(\forall V3y\_27\in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow(((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0l\in(ty\_2Elist\_2Elist\ A\_27a).(\forall V1f\in(A\_27b^{A\_27a}).((ap\ (c\_2Elist\_2ELENGTH\ A\_27b)\ (ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27a\ A\_27b)\ V1f)\ V0l))=(ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l)))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27b).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l1) = (ap\ (c\_2Elist\_2ELENGTH\ A\_27b)\ V1l2)) \Rightarrow (((ap\ (c\_2Elist\_2ELENGTH\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (c\_2Elist\_2EZIP\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Elist\_2Elist\ A\_27a)\ (ty\_2Elist\_2Elist\ A\_27b))\ V0l1)\ V1l2))) = (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l1)) \wedge ((ap\ (c\_2Elist\_2ELENGTH\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (c\_2Elist\_2EZIP\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Elist\_2Elist\ A\_27a)\ (ty\_2Elist\_2Elist\ A\_27b))\ V0l1)\ V1l2))) = (ap\ (c\_2Elist\_2ELENGTH\ A\_27b)\ V1l2)))))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27b).(\forall V2f1 \in (A\_27c^{A\_27a}).(\forall V3f2 \in (A\_27d^{A\_27b}).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l1) = (ap\ (c\_2Elist\_2ELENGTH\ A\_27b)\ V1l2)) \Rightarrow (((ap\ (c\_2Elist\_2EZIP\ A\_27c\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Elist\_2Elist\ A\_27c)\ (ty\_2Elist\_2Elist\ A\_27b))\ (ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27a\ A\_27c)\ V2f1)\ V0l1))\ V1l2)) = (ap\ (ap\ (c\_2Elist\_2EMAP\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)\ (ty\_2Epair\_2Eprod\ A\_27c\ A\_27b))\ (\lambda V4p \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27c\ A\_27b)\ (ap\ V2f1\ (ap\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b)\ V4p)))\ (ap\ (c\_2Epair\_2ESND\ A\_27a\ A\_27b)\ V4p))))\ (ap\ (c\_2Elist\_2EZIP\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Elist\_2Elist\ A\_27a)\ (ty\_2Elist\_2Elist\ A\_27b))\ V0l1)\ V1l2)))) \wedge ((ap\ (c\_2Elist\_2EZIP\ A\_27a\ A\_27d)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Elist\_2Elist\ A\_27a)\ (ty\_2Elist\_2Elist\ A\_27d))\ V0l1)\ (ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27b\ A\_27d)\ V3f2)\ V1l2))) = (ap\ (ap\ (c\_2Elist\_2EMAP\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27d))\ (\lambda V5p \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27d)\ (ap\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b)\ V5p))\ (ap\ V3f2\ (ap\ (c\_2Epair\_2ESND\ A\_27a\ A\_27b)\ V5p)))))\ (ap\ (c\_2Elist\_2EZIP\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Elist\_2Elist\ A\_27a)\ (ty\_2Elist\_2Elist\ A\_27b))\ V0l1)\ V1l2)))))))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27b).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l1) = (ap\ (c\_2Elist\_2ELENGTH\ A\_27b)\ V1l2)) \Rightarrow (\forall V2f \in ((A\_27c^{A\_27b})^{A\_27a}). ((ap\ (ap\ (ap\ (c\_2Elist\_2EMAP2\ A\_27c\ A\_27a\ A\_27b)\ V2f)\ V0l1)\ V1l2) = (ap\ (ap\ (c\_2Elist\_2EMAP\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)\ A\_27c)\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a\ A\_27b\ A\_27c)\ V2f))\ (ap\ (c\_2Elist\_2EZIP\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Elist\_2Elist\ A\_27a)\ (ty\_2Elist\_2Elist\ A\_27b))\ V0l1)\ V1l2))))))))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27b).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l1) = (ap\ (c\_2Elist\_2ELENGTH\ A\_27b)\ V1l2)) \Rightarrow (\forall V2f \in (((A\_27c^{A\_27b})^{A\_27a})^{A\_27c}). (\forall V3a \in A\_27c.((ap\ (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL2\ A\_27c\ A\_27a\ A\_27b)\ V2f)\ V3a)\ V0l1)\ V1l2) = (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)\ A\_27c)\ (\lambda V4a \in A\_27c.(ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a\ A\_27b\ A\_27c)\ (ap\ V2f\ V4a))))\ V3a)\ (ap\ (c\_2Elist\_2EZIP\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Elist\_2Elist\ A\_27a)\ (ty\_2Elist\_2Elist\ A\_27b))\ V0l1)\ V1l2)))))))))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0ls \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1f \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}). ((ap\ (ap\ (c\_2Elist\_2EMAP\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)\ A\_27b)\ V1f)\ (ap\ (c\_2Elist\_2EZIP\ A\_27a\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Elist\_2Elist\ A\_27a)\ (ty\_2Elist\_2Elist\ A\_27a))\ V0ls)\ V0ls))) = (ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27a\ A\_27b)\ (\lambda V2x \in A\_27a.(ap\ V1f\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V2x)\ V2x))))\ V0ls)))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y)) = V0x))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (c\_2Epair\_2ESND\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y)) = V1y))) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}\forall A\_27c.\ nonempty\ A\_27c{\Rightarrow}(\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}).(\forall V1x \in A\_27a.(\forall V2y \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V1x)\ V2y)) = (ap\ (ap\ V0f\ V1x)\ V2y)))))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}\forall A\_27c.\ nonempty\ A\_27c{\Rightarrow}(\forall V0P \in (A\_27c^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}).\ ((\lambda V1p \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(ap\ V0P\ V1p)) = (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a\ A\_27b\ A\_27c)\ (\lambda V2p1 \in A\_27a.(\lambda V3p2 \in A\_27b.(ap\ V0P\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2p1)\ V3p2))))))) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}\forall A\_27c.\ nonempty\ A\_27c{\Rightarrow}(\forall V0f \in ((A\_27a^{A\_27b})^{A\_27a}).(\forall V1e \in A\_27a.(\forall V2g \in (A\_27b^{A\_27c}).(\forall V3l \in (ty\_2Elist\_2Elist\ A\_27c).((ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27b\ A\_27a)\ V0f)\ V1e)\ (ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27c\ A\_27b)\ V2g)\ V3l)) = (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27c\ A\_27a)\ (\lambda V4x \in A\_27a.(\lambda V5y \in A\_27c.(ap\ (ap\ V0f\ V4x)\ (ap\ V2g\ V5y)))))\ V1e)\ V3l)))))) \tag{31}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow \forall A\_27e.nonempty\ A\_27e \Rightarrow (\forall V0f1 \in (((A\_27d^{A\_27c})^{A\_27b})^{A\_27a}).(\forall V1f2 \in (((A\_27e^{A\_27c})^{A\_27b})^{A\_27a}).(\forall V2bs \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V3cs \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27c)).(\forall V4a \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27d\ A\_27e). (((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V2bs) = (ap\ (c\_2Elist\_2ELENGTH\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27c))\ V3cs)) \Rightarrow ((ap\ (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL2\ (ty\_2Efinite\_\_map\_2Efmap\ A\_27d\ A\_27e)\ A\_27a\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27c))\ (\lambda V5fm \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27d\ A\_27e). (\lambda V6b \in A\_27a.(ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27b\ A\_27c\ (ty\_2Efinite\_\_map\_2Efmap\ A\_27d\ A\_27e))\ (\lambda V7c \in A\_27b.(\lambda V8d \in A\_27c.(ap\ (ap\ (c\_2Efinite\_\_map\_2EFUPDATE\ A\_27d\ A\_27e)\ V5fm)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27d\ A\_27e)\ (ap\ (ap\ (ap\ V0f1\ V6b)\ V7c)\ V8d))\ (ap\ (ap\ (ap\ V1f2\ V6b)\ V7c)\ V8d)))))))))\ V4a)\ V2bs)\ V3cs) = (ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFUPDATE\_\_LIST}\ A\_27d\ A\_27e)\ V4a)\ (ap\ (c\_2Elist\_2EZIP\ A\_27d\ A\_27e)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Elist\_2Elist\ A\_27d)\ (ty\_2Elist\_2Elist\ A\_27e))\ (ap\ (ap\ (ap\ (c\_2Elist\_2EMAP2\ A\_27d\ A\_27a\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27c))\ (\lambda V9b \in A\_27a.(ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27b\ A\_27c\ A\_27d)\ (ap\ V0f1\ V9b))))\ V2bs)\ V3cs))\ (ap\ (ap\ (ap\ (c\_2Elist\_2EMAP2\ A\_27e\ A\_27a\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27c))\ (\lambda V10b \in A\_27a.(ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27b\ A\_27c\ A\_27e)\ (ap\ V1f2\ V10b))))\ V2bs)\ V3cs)))))))))))$$