

thm_2Efinite_map_2EFOLDL_FUPDATE_LIST
 (TMZDQw-
 bc9Py5jkd82U8mKDzT7TnmEmpUE)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 6 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS_prod$

Let $ty_2Efinite_map_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efinite_map_2Efmap A0 A1) \tag{3}$$

Let $c_2Efinite_map_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2EFUPDATE\ A_27a\ A_27b \in (((ty_2Efinite_map_2E fmap\ A_27a\ A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})^{(ty_2Efinite_map_2EFUPDATE\ A_27a\ A_27b)})) \quad (4)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (5)$$

Let $c_2Elist_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EFOLDL\ A_27a\ A_27b \in (((A_27b^{(ty_2Elist_2Elist\ A_27a)})^{A_27b})^{((A_27b^{A_27a})^{A_27b})}) \quad (6)$$

Definition 7 We define $c_2Efinite_map_2EFUPDATE_LIST$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(ap\ (c_2Elist_2EFOLDL\ A_27a\ A_27b)\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ (c_2Efinite_map_2EFUPDATE\ A_27a\ A_27b)))$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EMAP\ A_27a\ A_27b \in (((ty_2Elist_2Elist\ A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{(A_27b^{A_27a})}) \quad (7)$$

Assume the following.

$$True \quad (8)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (9)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c.nonempty\ A_27c \Rightarrow (\forall V0f \in ((A_27a^{A_27b})^{A_27a}).(\forall V1e \in A_27a.(\forall V2g \in (A_27b^{A_27c}).(\forall V3l \in (ty_2Elist_2Elist\ A_27c).((ap\ (ap\ (ap\ (c_2Elist_2EFOLDL\ A_27b\ A_27a)\ V0f)\ V1e)\ (ap\ (c_2Elist_2EMAP\ A_27c\ A_27b)\ V2g)\ V3l)) = (ap\ (ap\ (ap\ (c_2Elist_2EFOLDL\ A_27c\ A_27a)\ (\lambda V4x \in A_27a.(\lambda V5y \in A_27c.(ap\ (ap\ V0f\ V4x)\ (ap\ V2g\ V5y)))))) V1e)\ V3l)))))) \quad (10)$$

Theorem 1

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c.nonempty\ A_27c \Rightarrow (\forall V0f1 \in (A_27b^{A_27a}).(\forall V1f2 \in (A_27c^{A_27a}).(\forall V2ls \in (ty_2Elist_2Elist\ A_27a).(\forall V3a \in (ty_2Efinite_map_2E fmap\ A_27b\ A_27c).((ap\ (ap\ (ap\ (c_2Elist_2EFOLDL\ A_27a\ (ty_2Efinite_map_2E fmap\ A_27b\ A_27c))\ (\lambda V4fm \in (ty_2Efinite_map_2E fmap\ A_27b\ A_27c).(\lambda V5k \in A_27a.(ap\ (ap\ (c_2Efinite_map_2EFUPDATE\ A_27b\ A_27c)\ V4fm)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27b\ A_27c)\ (ap\ V0f1\ V5k))\ (ap\ V1f2\ V5k)))))) V3a)\ V2ls) = (ap\ (ap\ (c_2Efinite_map_2EFUPDATE_LIST\ A_27b\ A_27c)\ V3a)\ (ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ (ty_2Epair_2Eprod\ A_27b\ A_27c))\ (\lambda V6k \in A_27a.(ap\ (ap\ (c_2Epair_2E_2C\ A_27b\ A_27c)\ (ap\ V0f1\ V6k))\ (ap\ V1f2\ V6k)))))) V2ls)))))) \quad (11)$$