

thm_2Efinite_map_2EFUPDATE_LIST_APPLY_NOT_MEM (TMXCwQqdY1X8Widn939divoXo6en2kfds8M)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_2Emin_2E_3D (2^{A_27a}))$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $ty_2Efinite_map_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efinite_map_2Efmap\ A0\ A1) \tag{3}$$

Let $c_2Efinite_map_2Efmap_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2Efmap_REP \\ & A_27a\ A_27b \in (((ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a})^{(ty_2Efinite_map_2Efmap\ A_27a\ A_27b)}) \end{aligned} \quad (4)$$

Let $c_2Esum_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EOUTL \\ & A_27a\ A_27b \in (A_27a)^{(ty_2Esum_2Esum\ A_27a\ A_27b)} \end{aligned} \quad (5)$$

Definition 9 We define $c_2Efinite_map_2EFAPPLY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod \\ & A0\ A1) \end{aligned} \quad (6)$$

Let $c_2Efinite_map_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2EFUPDATE \\ & A_27a\ A_27b \in (((ty_2Efinite_map_2Efmap\ A_27a\ A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})^{(ty_2Efinite_map$$
 \end{aligned} \quad (7)

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (8)$$

Let $c_2Elist_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EFOLDL \\ & A_27a\ A_27b \in (((A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{A_27b})^{((A_27b)^{A_27a})^{A_27b}} \end{aligned} \quad (9)$$

Definition 10 We define $c_2Efinite_map_2EFUPDATE_LIST$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (ap\ (c_2Elist_2E$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EMAP \\ & A_27a\ A_27b \in (((ty_2Elist_2Elist\ A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{(A_27b)^{A_27a}}) \end{aligned} \quad (10)$$

Let $c_2Elist_2ELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELIST_TO_SET\ A_27a \in \\ & ((2^{A_27a})^{(ty_2Elist_2Elist\ A_27a)}) \end{aligned} \quad (11)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist \\ & A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \end{aligned} \quad (12)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (13)$$

Definition 11 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (14)$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (15)$$

Definition 13 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF).$

Definition 14 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E$

Definition 15 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 16 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \quad (16)$$

Definition 17 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap\ (c_2$

Assume the following.

$$True \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (20)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B)))))) \quad (24)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (25)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A_27a.(\forall V3x_27 \in A_27a.(\forall V4y \in A_27a.(\forall V5y_27 \in A_27a.(((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge ((\neg(p V1Q) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND \ A_27a) V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND \ A_27a) V1Q) V3x_27) V5y_27)))))) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow ((\forall V0t1 \in A_27a.(\forall V1t2 \in A_27a.((ap (ap (ap (c_2Ebool_2ECOND \ A_27a) c_2Ebool_2ET) V0t1) V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a.(\forall V3t2 \in A_27a.((ap (ap (ap (c_2Ebool_2ECOND \ A_27a) c_2Ebool_2EF) V2t1) V3t2) = V3t2)))) \quad (28)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0f \in (ty_2Efinite_map_2Efmap\ A.27a\ A.27b).(\forall V1a \in \\
& \quad A.27a.(\forall V2b \in A.27b.(\forall V3x \in A.27a.((ap\ (ap\ (c_2Efinite_map_2EFAPPLY \\
& \quad A.27a\ A.27b)\ (ap\ (ap\ (c_2Efinite_map_2EFUPDATE\ A.27a\ A.27b)\ V0f) \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27b)\ V1a)\ V2b)))\ V3x) = (ap\ (ap\ (ap \\
& \quad (c_2Ebool_2ECOND\ A.27b)\ (ap\ (ap\ (c_2Emin_2E_3D\ A.27a)\ V3x)\ V1a)) \\
& \quad V2b)\ (ap\ (ap\ (c_2Efinite_map_2EFAPPLY\ A.27a\ A.27b)\ V0f)\ V3x)))))) \\
& \hspace{15em} (29)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0f \in (ty_2Efinite_map_2Efmap\ A.27a\ A.27b).(((ap\ (ap \\
& \quad (c_2Efinite_map_2EFUPDATE_LIST\ A.27a\ A.27b)\ V0f)\ (c_2Elist_2ENIL \\
& \quad (ty_2Epair_2Eprod\ A.27a\ A.27b))) = V0f) \wedge (\forall V1h \in (ty_2Epair_2Eprod \\
& \quad A.27a\ A.27b).(\forall V2t \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod \\
& \quad A.27a\ A.27b)).((ap\ (ap\ (c_2Efinite_map_2EFUPDATE_LIST\ A.27a \\
& \quad A.27b)\ V0f)\ (ap\ (ap\ (c_2Elist_2ECONS\ (ty_2Epair_2Eprod\ A.27a\ A.27b)) \\
& \quad V1h)\ V2t)) = (ap\ (ap\ (c_2Efinite_map_2EFUPDATE_LIST\ A.27a\ A.27b) \\
& \quad (ap\ (ap\ (c_2Efinite_map_2EFUPDATE\ A.27a\ A.27b)\ V0f)\ V1h))\ V2t)))))) \\
& \hspace{15em} (30)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad (\forall V0f \in (A.27b^{A.27a}).((ap\ (ap\ (c_2Elist_2EMAP\ A.27a\ A.27b) \\
& \quad V0f)\ (c_2Elist_2ENIL\ A.27a)) = (c_2Elist_2ENIL\ A.27b)) \wedge (\forall V1f \in \\
& \quad (A.27b^{A.27a}).(\forall V2h \in A.27a.(\forall V3t \in (ty_2Elist_2Elist \\
& \quad A.27a).((ap\ (ap\ (c_2Elist_2EMAP\ A.27a\ A.27b)\ V1f)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad A.27a)\ V2h)\ V3t)) = (ap\ (ap\ (c_2Elist_2ECONS\ A.27b)\ (ap\ V1f\ V2h)) \\
& \quad (ap\ (ap\ (c_2Elist_2EMAP\ A.27a\ A.27b)\ V1f)\ V3t)))))) \\
& \hspace{15em} (31)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0h \in A.27b.(\forall V1t \in (ty_2Elist_2Elist\ A.27b).((\\
& \quad (ap\ (c_2Elist_2ELIST_TO_SET\ A.27a)\ (c_2Elist_2ENIL\ A.27a)) = \\
& \quad (c_2Epred_set_2EEMPTY\ A.27a)) \wedge ((ap\ (c_2Elist_2ELIST_TO_SET \\
& \quad A.27b)\ (ap\ (ap\ (c_2Elist_2ECONS\ A.27b)\ V0h)\ V1t)) = (ap\ (ap\ (c_2Epred_set_2EINSERT \\
& \quad A.27b)\ V0h)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A.27b)\ V1t)))))) \\
& \hspace{15em} (32)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A.27a)}). \\
& \quad (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A.27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\
& \quad A.27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A.27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\
& \quad c_2Elist_2ECONS\ A.27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\
& \quad A.27a).(p\ (ap\ V0P\ V3l)))))) \\
& \hspace{15em} (33)
\end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0x \in (ty_2Epair_2Eprod\ A_27a\ A_27b). (\exists V1q \in A_27a. \\ & (\exists V2r \in A_27b. (V0x = (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b) \\ & V1q)\ V2r)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0x \in A_27a. (\forall V1y \in A_27b. ((ap\ (c_2Epair_2EFST\ A_27a \\ & A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V0x))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg (p\ (ap\ (ap \\ & (c_2Ebool_2EIN\ A_27a)\ V0x)\ (c_2Epred_set_2EEMPTY\ A_27a)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ & A_27a. (\forall V2s \in (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\ & V0x)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V1y)\ V2s)))) \Leftrightarrow ((V0x = \\ & V1y) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ V2s)))))) \end{aligned} \quad (37)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0kvl \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b)). \\ & (\forall V1f \in (ty_2Efinite_map_2E fmap\ A_27a\ A_27b). (\forall V2k \in \\ & A_27a. ((\neg (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2k)\ (ap\ (c_2Elist_2ELIST_TO_SET \\ & A_27a)\ (ap\ (ap\ (c_2Elist_2EMAP\ (ty_2Epair_2Eprod\ A_27a\ A_27b) \\ & A_27a)\ (c_2Epair_2EFST\ A_27a\ A_27b))\ V0kvl)))))) \Rightarrow ((ap\ (ap\ (c_2Efinite_map_2EFAPPLY \\ & A_27a\ A_27b)\ (ap\ (ap\ (c_2Efinite_map_2EFUPDATE_LIST\ A_27a\ A_27b) \\ & V1f)\ V0kvl))\ V2k) = (ap\ (ap\ (c_2Efinite_map_2EFAPPLY\ A_27a\ A_27b) \\ & V1f)\ V2k)))))) \end{aligned}$$